Feb 20, 2017

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Announcements

HW #5 :	2.3	due 2/24. Friday
HW #6	2.4	
	2.5	due 3/2 Thursday
	2.6	

MT 1 March 3. Friday

Review Session Thursday March 2 evening

Grade Scale for Exams: MT1, MT2, Final

These cuts may be lowered for some exams depending on many factors. However, They will not go up, for any exam.

A−, A

90%

B−, B, B+

80%

C−, C, C+

65%

D−, D, D+

50%

F

For example: In an exam: 81% will be B− or higher, 81% will never be C+ or lower.

Caution: HW, & quizzes are subject to different cuts.

2.4

$$f_x = \frac{\partial f}{\partial x}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\, \partial x}$$

$\frac{\partial}{\partial x}$ calculated first, $\frac{\partial}{\partial y}$ calculated next.

<u>Thm:</u> If $f : X^{\text{open}} \subseteq \mathbb{R}^2 \to \mathbb{R}$,

if f has continuous first & second order partial derivatives, then

$$\frac{\partial^2 f}{\partial x\, \partial y} = \frac{\partial^2 f}{\partial y\, \partial x}$$

<u>CAUTION</u>: This is not always true.

<u>See:</u> Exc #30 p 142

Ex

$$f(x,y,z) = x^2y^3 + y^2z$$

Find all first & second order derivatives of f

$$f_x = 2xy^3$$

$$f_y = 3x^2y^2 + 2yz$$

$$f_z = y^2$$

First Derivative matrix

$$Df = \begin{bmatrix} 2xy^3 & 3x^2y^2 + 2yz & y^2 \end{bmatrix}$$

$$\nabla f = (2xy^3,\ 3x^2y^2 + 2yz,\ y^2)$$

Gradient

$f_{xx} = 2y^3$ $\quad f_{xy} = 6xy^2$ $\quad f_{xz} = 0$

$f_{yx} = 6xy^2$ $\quad f_{yy} = 6x^2y + 2z$ $\quad f_{yz} = 2y$

$f_{zx} = 0$ $\quad f_{zy} = 2y$ $\quad f_{zz}$ 0

$$D(\nabla f) = Hf = \begin{bmatrix} 2y^3 & 6xy^2 & 0 \\ 6xy^2 & 6x^2y + 2z & 2y \\ 0 & 2y & 0 \end{bmatrix} = \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{bmatrix}$$

Hessian Matrix

2nd Derivative matrix.

2.5 CHAIN RULE

Review (Calculus I)

$n=1$

$$\begin{array}{ccccc} I & \xrightarrow{g} & J & \xrightarrow{f} & \mathbb{R}^1 \\ \cap\!| & & \cap\!| & & y \\ \mathbb{R}^1 & & \mathbb{R}^1 & & \\ u & & x & & \end{array}$$

$$x = g(u)$$

$$y = f(x)$$

$$y = f(g(u)) = (f \circ g)(u)$$

$$\frac{dy}{du} = \frac{dy}{dx} \cdot \frac{dx}{du}$$

$$\frac{d}{du}(f \circ g)(u) = f'(g(u)) \cdot g'(u)$$

$$= f'(x) \cdot g'(u)$$

THEOREM Let $U^{\text{open}} \xrightarrow{G} V^{\text{open}} \xrightarrow{F} \mathbb{R}^k$, with $U \subseteq \mathbb{R}^n$, $V \subseteq \mathbb{R}^m$, and $a \in U$.

Let G be diffble at a, and

Let F be diffble at $G(a)$.

Then: $F \circ G$ is diffble at a, and

$$\underbrace{D(F \circ G)(a)}_{k \times n} = \underbrace{DF(G(a))}_{k \times m} \cdot \underbrace{DG(a)}_{m \times n}$$

Exc #28 p 156

$$\text{given}\begin{cases} g: \mathbb{R}^3 \to \mathbb{R}^2 \\ g(1,-1,3) = (2,5) \\ Dg(1,-1,3) = \begin{bmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{bmatrix} \\ \\ f: \mathbb{R}^2 \to \mathbb{R}^2 \\ f(x,y) = (2xy,\ 3x - y + 5) \end{cases}$$

Want:

$$? = D(f\circ g)(1,-1,3)) = Df(\underbrace{g(1,-1,3)}_{(2,5)}) \cdot D_g(1,-1,3)$$

$$\underset{2\times 2}{Df} = \begin{bmatrix} 2y & 2x \\ 3 & -1 \end{bmatrix}$$

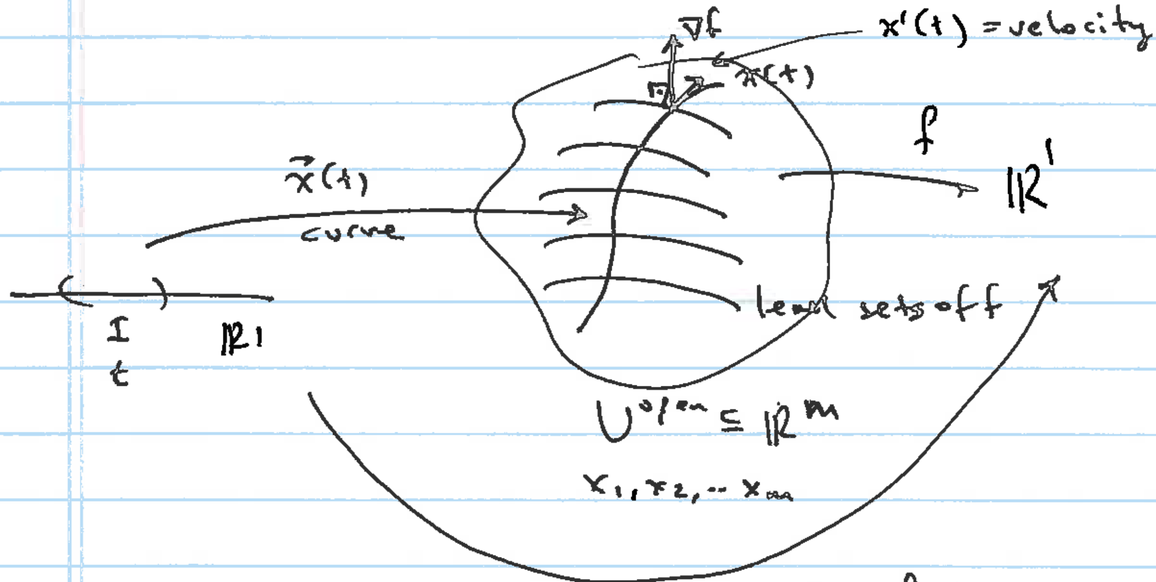
$$(Df)(2,5) = \begin{bmatrix} 10 & 4 \\ 3 & -1 \end{bmatrix}$$

$$D(f\circ g)(1,-1,3) = \begin{bmatrix} 10 & 4 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 4 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & -10 & 28 \\ -1 & -3 & -7 \end{bmatrix} \quad \#.$$

Closed Form: $D(F \circ G)(a) = DF(G(a)) \cdot DG(a)$

Open Form: Crucial Case:



$$f(x(t)) = f(x_1(t), x_2(t), \ldots x_m(t))$$

$$D(f(\vec{x}(t)) = Df(\vec{x}(t)) \cdot D\vec{x}(t)$$

$$\underset{1 \times 1}{\left[\frac{d}{dt} f(x(t))\right]} = \underset{1 \times m}{\left[\frac{\partial f}{\partial x_1} \ \frac{\partial f}{\partial x_2} \ \cdots \ \frac{\partial f}{\partial x_m}\right]} \cdot \underset{m \times 1}{\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \vdots \\ \frac{dx_m}{dt} \end{bmatrix}}$$

matrix multiplication

$$\left[\frac{d}{dt} f(x(t))\right] = \underbrace{\left[\frac{\partial f}{\partial x_1} \cdot \frac{dx_1}{dt} + \frac{\partial f}{\partial x_2} \cdot \frac{dx_2}{dt} + \cdots \frac{\partial f}{\partial x_m} \cdot \frac{dx_m}{dt}\right]}_{\underbrace{\nabla f(x(t))}_{\text{gradient}} \cdot \underbrace{x'(t)}_{\text{velocity}}}$$

Dot product