Feb 13, 2017

1

2.3

Defn Let $f : X \subseteq \mathbb{R}^n \longrightarrow \mathbb{R}$

$x_1, x_2 \ldots x_n$

$f = f(x_1, x_2, \ldots x_n)$.

Let $a_1, a_2 \ldots a_n \in \mathbb{R}$

$f(\underbrace{a_1, a_2, \ldots a_{i-1}}_{\text{fix}}, x_i, \underbrace{a_{i+1}, \ldots a_n}_{\text{fix}})$ is a partial function.

leave it as a variable

Defn

$$\frac{\partial f}{\partial x_i}(a_1, a_2 \ldots a_n) = \frac{d}{dx_i} f(a_1, a_2, \ldots a_{i-1}, x_i, a_{i+1}, \ldots a_n) \Big|_{x_i = a_i}$$

is called the partial derivative of f with respect to x_i at $(a_1, a_2, \ldots a_n)$ provided that both the partial function & its derivative exist.

Other Notation: $\dfrac{\partial f}{\partial x_i} = D_{x_i} f = f_{x_i}$

Ex $f(x,y) = x^2y - 5e^x y^3 + y$

Calculate $\frac{\partial f}{\partial x}(0,-1)$

$\frac{\partial f}{\partial y}(0,-1)$

$$\frac{\partial f}{\partial x}(0,-1) = \frac{d}{dx} f(x,\underset{y}{\underset{\shortparallel}{-1}})\Big|_{x=0}$$

$$= \frac{d}{dx}\left(-x^2 + 5e^x - 1\right)\Big|_{x=0}$$

$$= \left(-2x + 5e^x + 0\right)\Big|_{x=0} = \left(-2\cdot 0 + 5\cdot e^0 + 0\right)$$

$$= 5$$

$$\frac{\partial f}{\partial y}(0,-1) = \frac{d}{dy} f(\underset{x}{0},y)\Big|_{y=-1}$$

$$= \frac{d}{dy}\left(0 - 5y^3 + y\right)\Big|_{y=-1} = -15y^2 + 1\Big|_{y=-1}$$

$$= -14.$$

I $f(x,y) = x^2y - 5e^x y^3 + y$

$$\frac{\partial f}{\partial x} = 2x \cdot y - 5e^x y^3 + 0$$

Can consider all y's as constants.

$$\frac{\partial f}{\partial x}(0,-1) = 2 \cdot 0 \cdot -1 - 5 \cdot e^0 \cdot (-1)^3 + 0$$

$$= 5$$

$$\frac{\partial f}{\partial y} = x^2 - 3y^2 \cdot 5e^x + 1$$

Can consider all x's as constants

$$\frac{\partial f}{\partial y}(0,-1) = 0 - 3(-1)^2 \cdot 5 \cdot e^0 + 1 = -14$$

Ex

$$f(x,y) = e^{3x^2+y^3}$$

$$\frac{\partial f}{\partial x} = 6x\,e^{3x^2+y^3}$$

$$\frac{\partial f}{\partial y} = 3y^2 \cdot e^{3x^2+y^3}$$

Ex

$$F(x,y,z) = 3x^2 + y^3x + 6$$

$$\frac{\partial F}{\partial x} = 6x + y^3 + 0$$

$$\frac{\partial F}{\partial y} = 3y^2x$$

$$\frac{\partial F}{\partial z} = 0$$

Ex

$$f(x,y) = (x^2+xy)(x^3y + y^2 + 6x)$$

$$\frac{\partial f}{\partial x} = (x^2+xy)(3x^2y+6) + (2x+y)(x^3y+y^2+6x)$$

$$\frac{\partial f}{\partial y} = (x^2+xy)(x^3+2y) + \quad x \quad (x^3y+y^2+6x)$$

Ex:

$$g(x,y) = \frac{x^3 + y^2 - xy}{x - 2y}$$

$$\frac{\partial g}{\partial x} = \frac{(3x^2 - y)(x - 2y) - 1 \cdot (x^3 + y^2 - xy)}{(x - 2y)^2}$$

$$\frac{\partial g}{\partial y} = \frac{(2y - x) \cdot (x - 2y) - (x^3 + y^2 - xy) \cdot (-2)}{(x - 2y)^2}$$

Exc #6 $f(x,y) = \ln(x^2 + y^2)$

$$\frac{\partial f}{\partial x} = f_x = \frac{2x}{x^2 + y^2}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2}$$

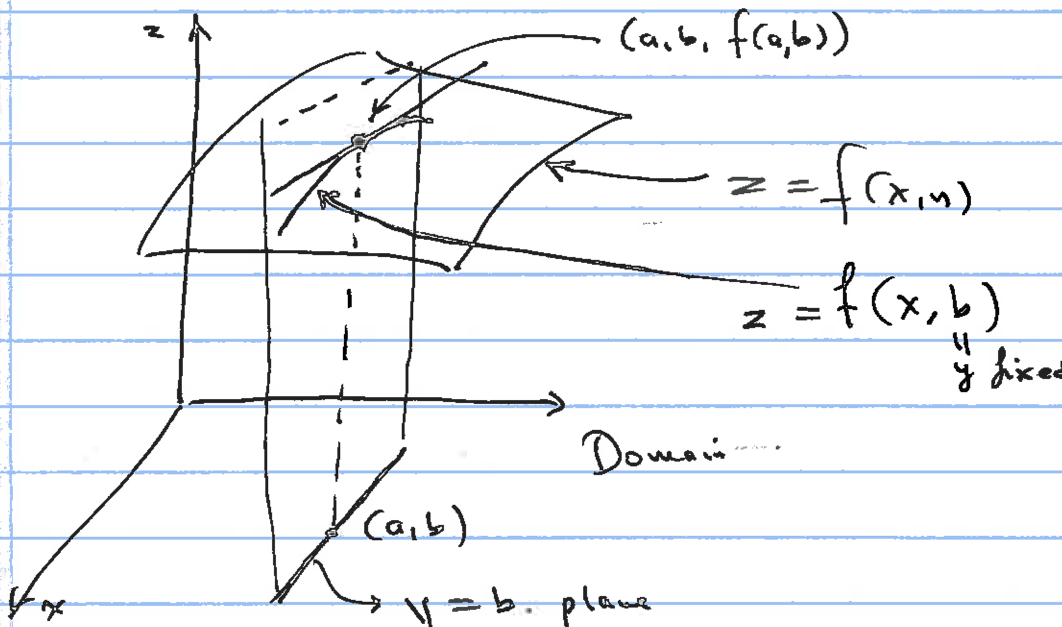
Ex

$$h = \cos(\ln(xy^2 - y))$$

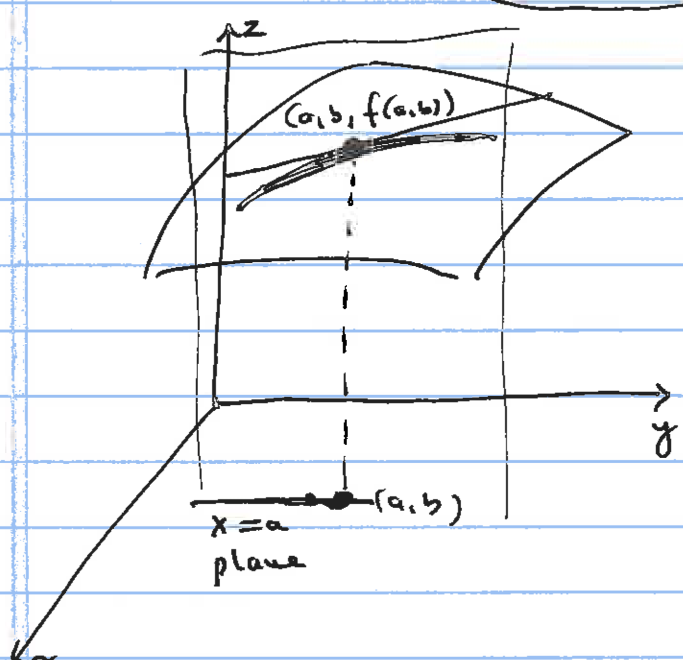
$$\frac{\partial h}{\partial x} = -\sin(\ln(xy^2 - y)) \cdot \frac{1 \cdot y^2 - 0}{xy^2 - y}$$

$$\frac{\partial h}{\partial y} = -\sin(\ln(xy^2 - y)) \cdot \frac{2xy - 1}{xy^2 - y}$$

Geometric meaning of $\frac{\partial f}{\partial x}(a,b)$, $\frac{\partial f}{\partial y}(a,b)$



The slope of the tangent line to the curve of intersection of the graph of $z=f(x,y)$ with plane $y=b$ at $(a,b,f(a,b))$ is $\frac{\partial f}{\partial x}(a,b) = \left(\frac{d}{dx} f(x,b)\right|_{x=a}$



The slope of the tangent line to the curve of intersection of the graph of $z=f(x,y)$ with the plane $x=a$ at $(a,b,f(a,b))$ is

$$\frac{\partial f}{\partial y}(a,b) = \left(\frac{d}{dy} f(a,y)\right|_{y=b}$$

Ex $h(x,y) = |x|$.

$$\frac{\partial h}{\partial x}(0,0) = \text{DNE}$$

$$\frac{\partial h}{\partial y}(0,0) = \left(\frac{d}{dy} h(0,y) \Big|_{y=0} \right)$$

$$= \frac{d}{dy} 0 = 0.$$