2.1 Ex

$$f(x,y,z) = x-y+3z : \underset{x,y,z}{\mathbb{R}^3} \longrightarrow \underset{w}{\mathbb{R}^1}$$

How? → (Explicit) graph $\subseteq \mathbb{R}^4 \ (x,y,z,w)$

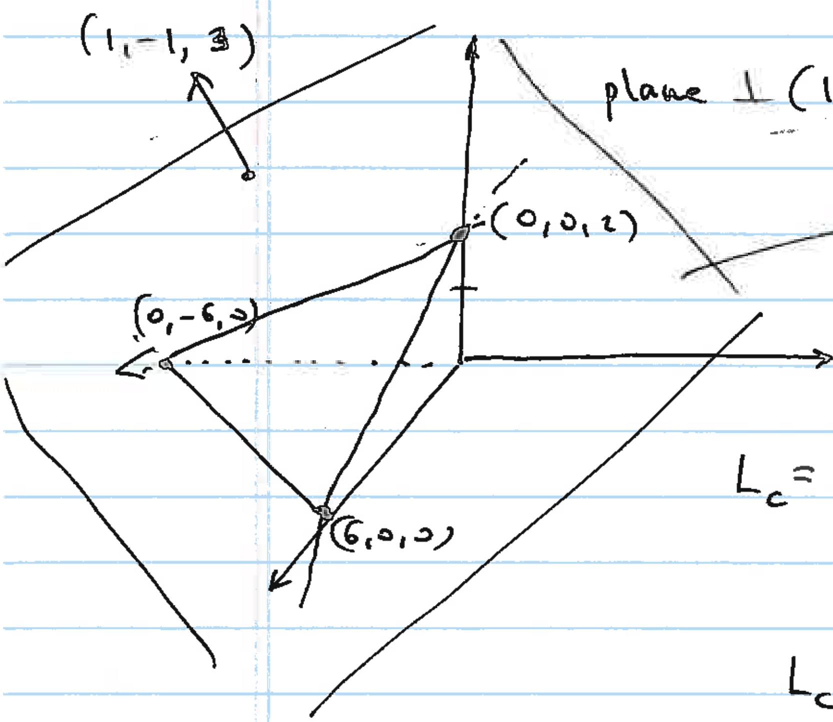
Best to use : Implicit graph $\subseteq \mathbb{R}^3 \ (x,y,z)$

not informative → Parametric graph $\subseteq \mathbb{R}^1 (w)$

$$\{(x,y,z) \mid f(x,y,z) = x-y+3z = c\}$$

$\{(x,y,z) \mid x-y+3z=6\}$ Take $c=6$ first.

plane $\perp$ $(1,-1,3)$



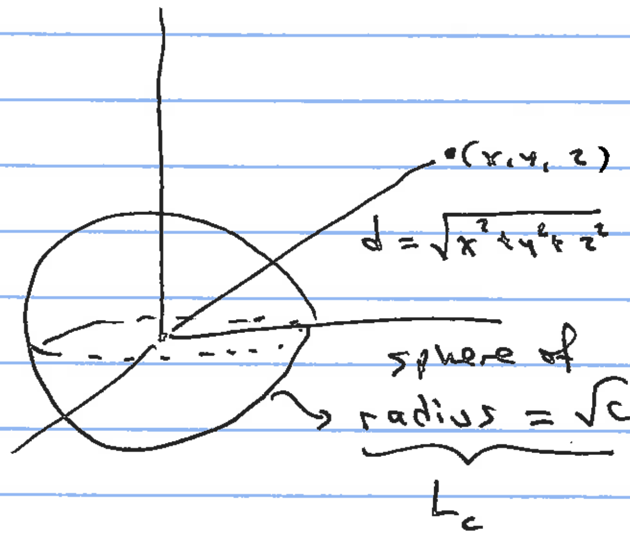
plane $\perp (1,-1,3)$ thru $(6,0,0)$

$\xrightarrow{f} 6$

As c varies
$L_c = \{(x,y,z) \mid x-y+3z=c\}$
is a family of planes
all $\perp$ $(1,-1,3)$; and
L_c contains $(c,0,0)$, $(0,-c,0)$, $(0,0,\frac{c}{3})$

Ex 2 $\quad f(x,y,z) = x^2+y^2+z^2 : \mathbb{R}^3 \longrightarrow \mathbb{R}^1$

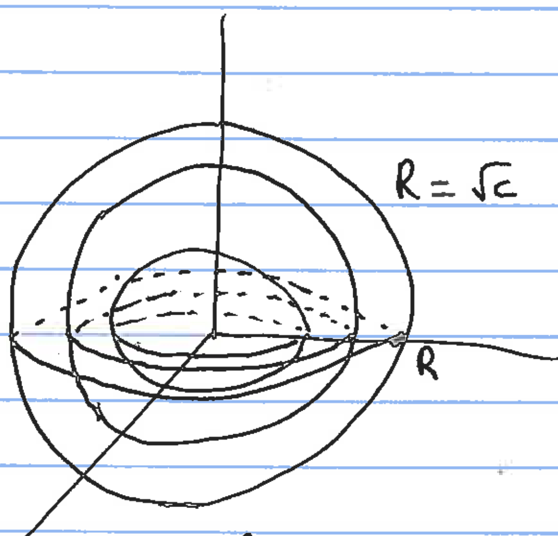
$L_c = \{(x,y,z) \mid x^2+y^2+z^2 = c\}$



when $c > 0$ $= L_c$

$\{(0,0,0)\} = L_0$

$\emptyset = L_c$ if $c < 0$.

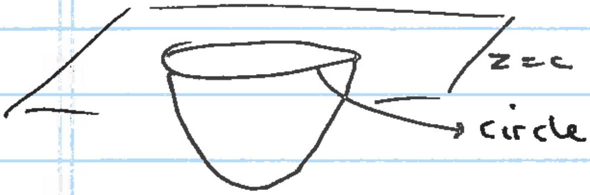


$R = \sqrt{c}$ when $c \geq 0$.

L_c is A family of spheres centered at origin, with radius $\sqrt{c}$, when $c > 0$

Important surfaces (quadratic)

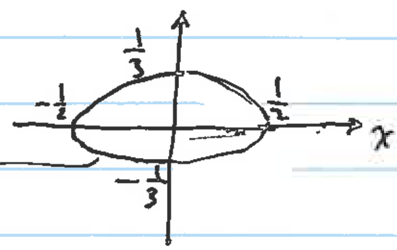
(i) $z = x^2+y^2$



circular paraboloid

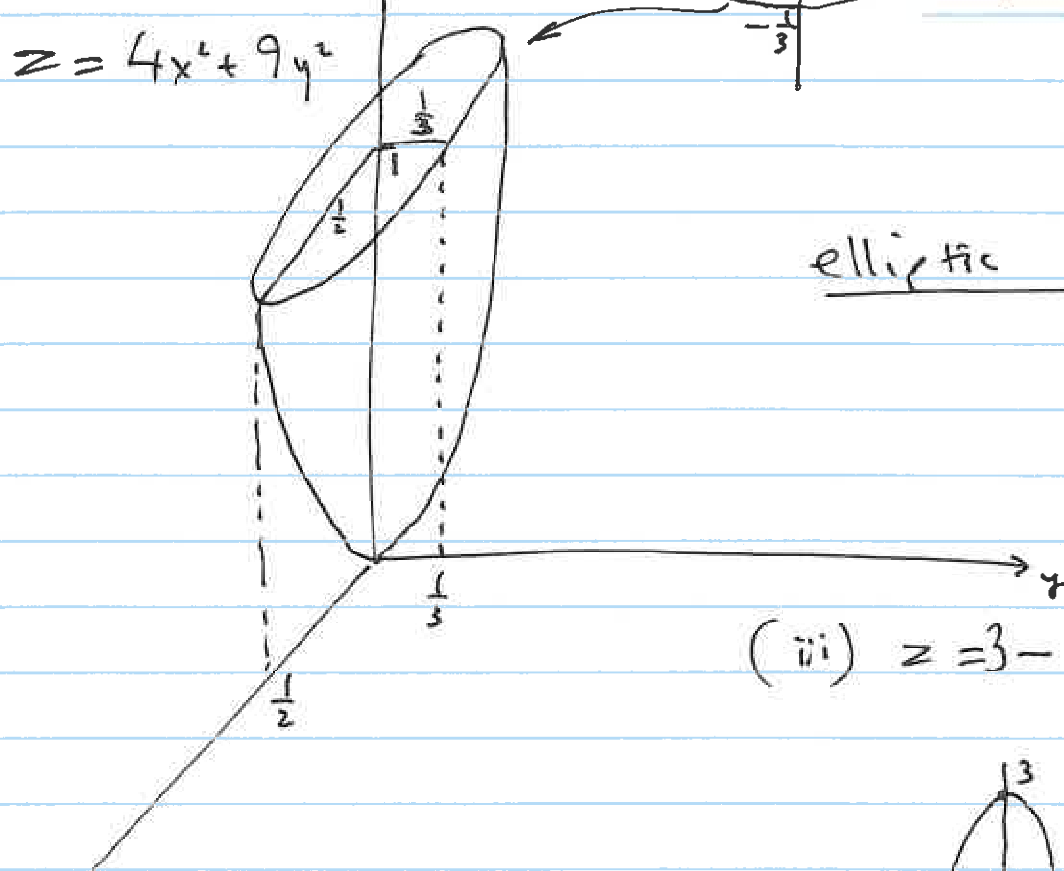
(ii) $z = 4x^2+9y^2$

If $z=1$,
$4x^2+9y^2=1$ is an ellipse



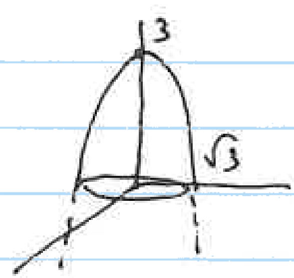
in the $z=1$ plane

$z=4x^2+9y^2$



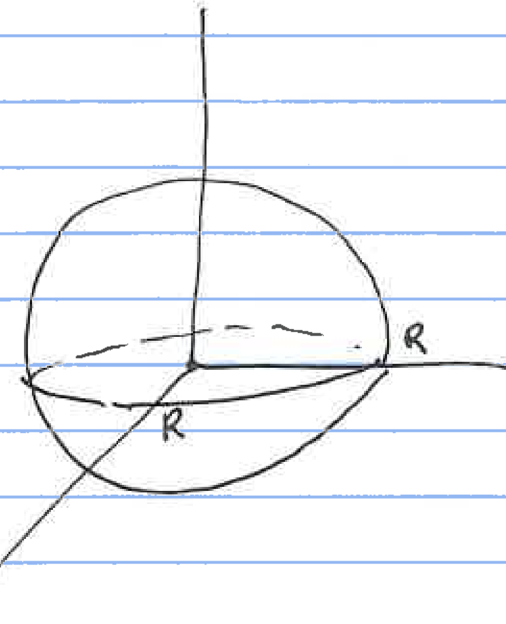
elliptic paraboloid

(iii) $z = 3-(x^2+y^2)$

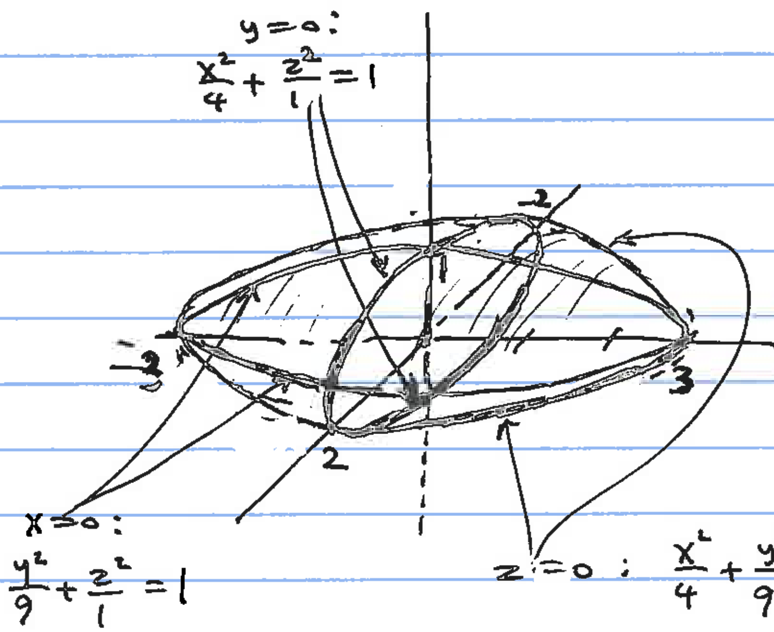


Spheres, Ellipsoids

(i) $x^2+y^2+z^2=R^2$



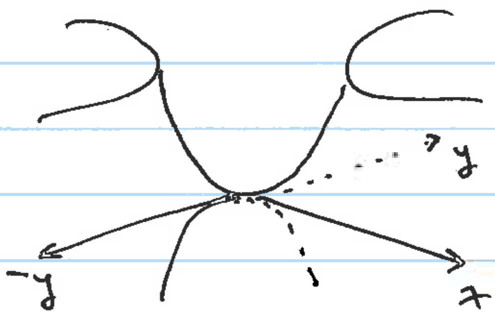
(ii) $\dfrac{x^2}{4}+\dfrac{y^2}{9}+\dfrac{z^2}{1}=1$. ellipsoid



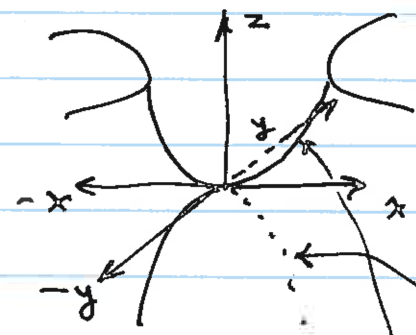
Each intersection with $\left.\begin{matrix} xy \\ xz \\ yz \end{matrix}\right\}$ planes are ellipses

Saddles:

$z = 2xy$



$z = x^2 - y^2 = f(x,y)$



Intersections with

- xz plane $\leftarrow$ $f(x,0) = x^2$
- yz plane $\leftarrow$ $f(0,y) = -y^2$

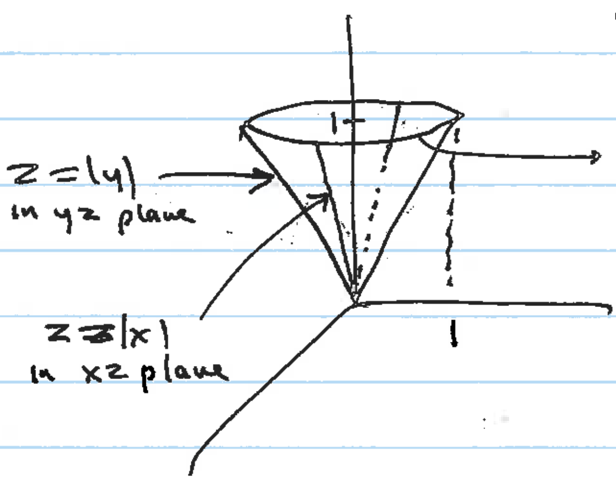
Cones:

(i) $z = \sqrt{x^2+y^2} = g(x,y)$

$g(x,0) = \sqrt{x^2} = |x|$

$g(0,y) = \sqrt{y^2} = |y|.$

$1 = z = \sqrt{x^2+y^2}$



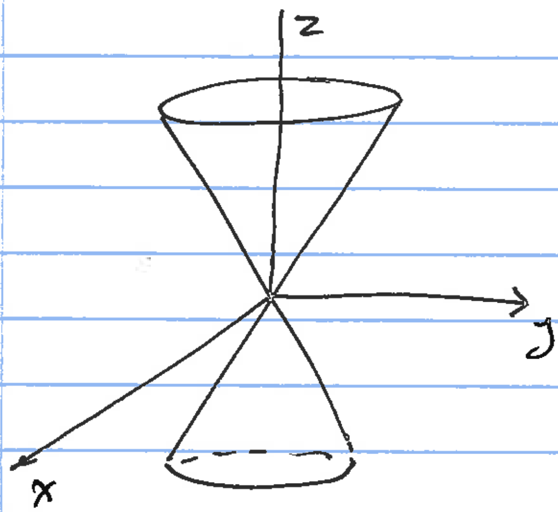
$\sqrt{x^2+y^2} = 1$

(ii) $x^2+y^2-z^2=0$

$$z^2 = x^2+y^2$$

$$z = \pm\sqrt{x^2+y^2}$$

Double Cone.



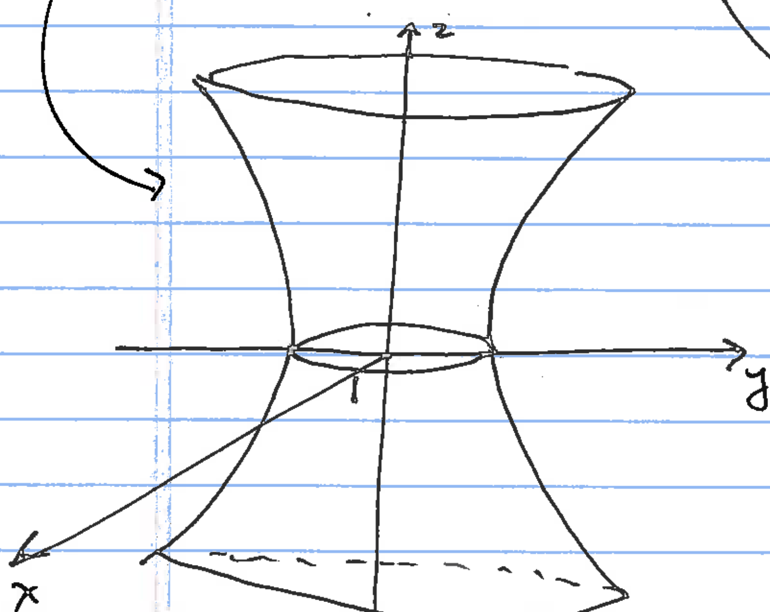
Hyperboloids:

$x^2+y^2-z^2=1$

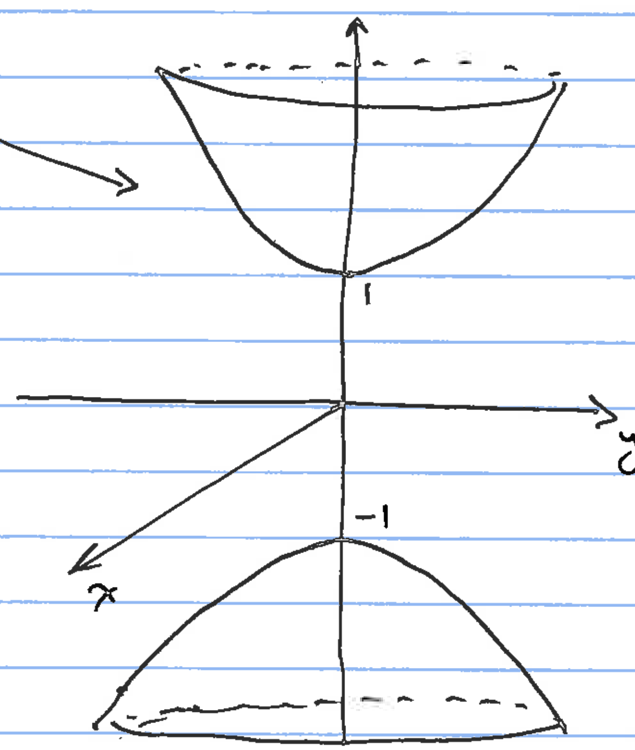
$x^2+y^2-z^2=-1$

How do you know which is which?

Put $z=0$ to find the intersection with xy plane



one sheeted



Two sheeted