Example 1 $f(x,y) = x^2 + y^2 - 1 : \mathbb{R}^2 \to \mathbb{R}^1$

Given a number $c \in \mathbb{R}$ = codomain

what is

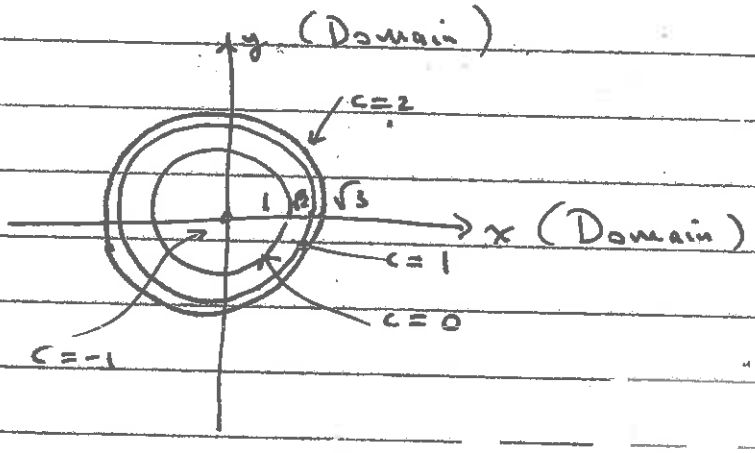
$$f^{-1}(c) = L_c = \{(x,y) \mid f(x,y) = c\}$$

L_c: level set for value c / contour curve

$$L_c = \{(x,y) \mid x^2+y^2-1 = c\}$$
$$= \{(x,y) \mid x^2+y^2 = 1+c\}$$

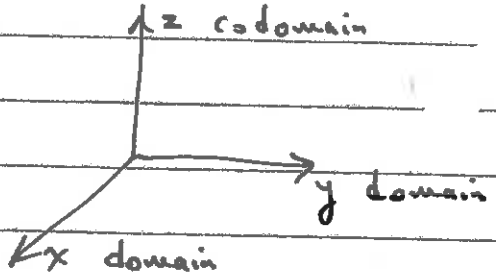
- $\emptyset$ if $c < -1$.
- $\{(0,0)\}$ if $c = -1$
- circle of radius $\sqrt{1+c}$ if $c > -1$.

Level sets:



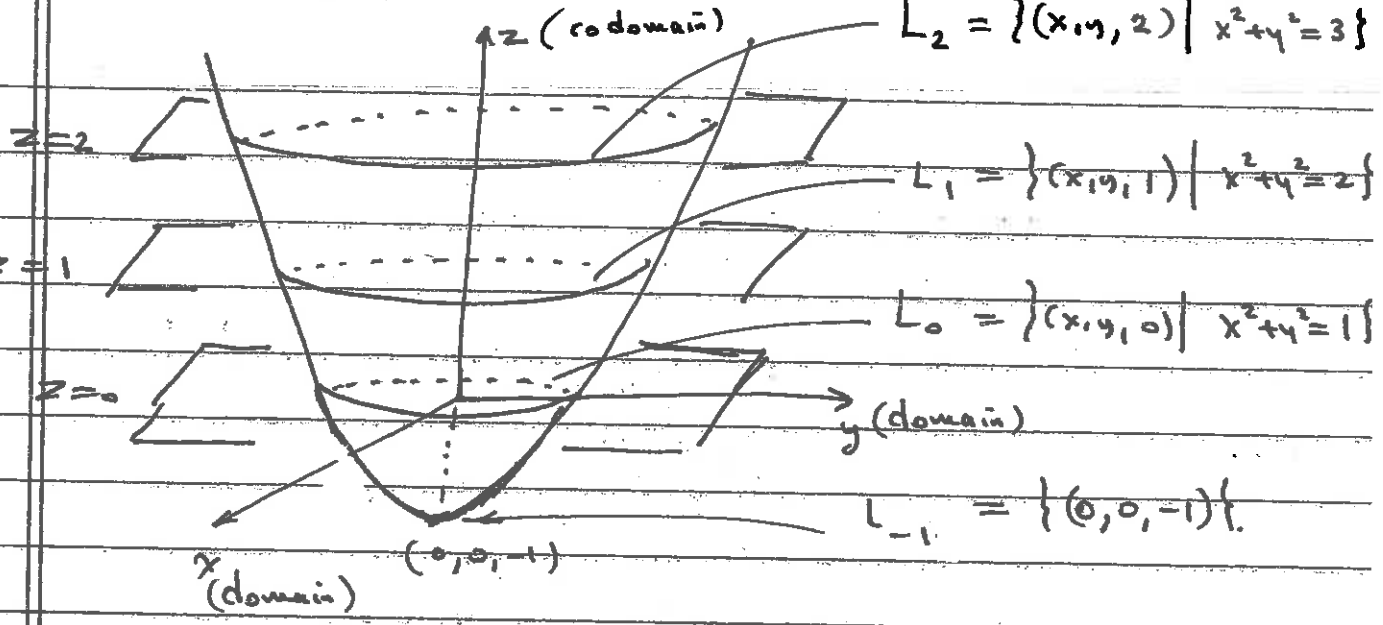
What is the graph of f?

① it lies in $\mathbb{R}^2 \times \mathbb{R}^1 = \mathbb{R}^3$



② Horizontal slices by planes $z = c$ are L_c

graph of f



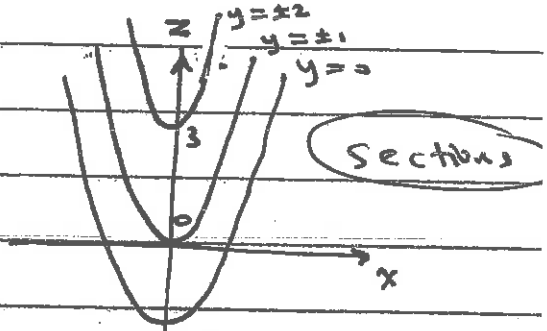
Sections: are obtained by restricting some domain variables to constants:

Continue

$f(x,y) = x^2+y^2-1$

or

$x=0$	$f(0,y) = y^2-1$	$y=0$	$f(x,0) = x^2-1$
$x=\pm 1$	$f(\pm 1,y) = y^2$	$y=\pm 1$	$f(x,1) = x^2$
$x=\pm 2$	$f(2,y) = 3+y^2$	$y=\pm 2$	$f(x,2) = 3+x^2$

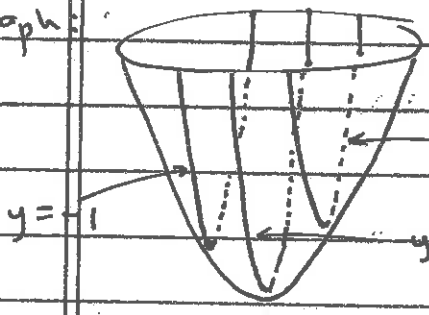


Sections

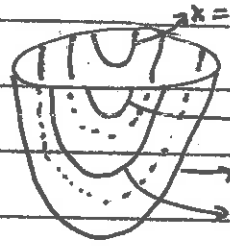
These are the slices of the graph with the planes

$x = -2, -1, 0, 1, 2$

Graph:



y=c sections



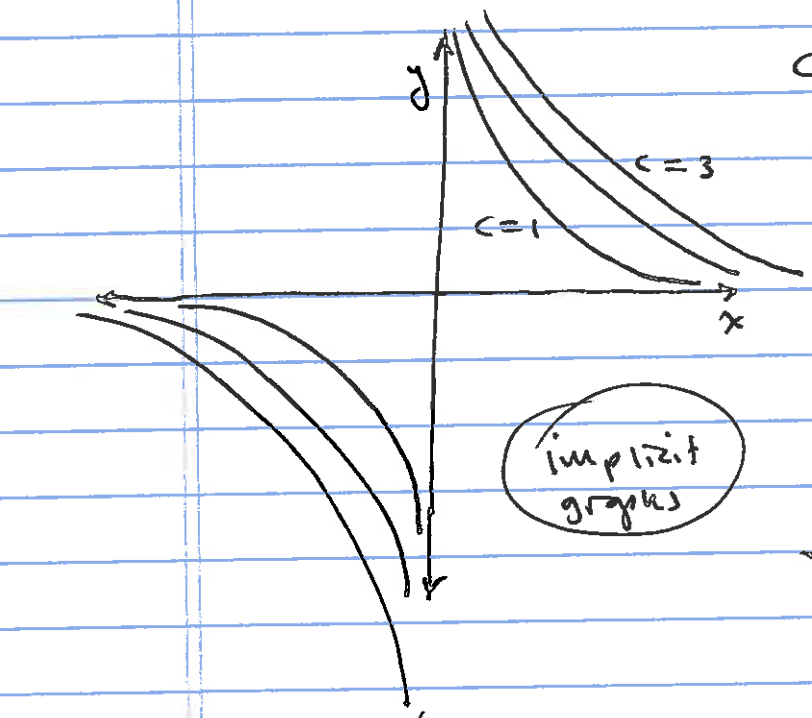
x=c sections

Ex 2 $f(x,y) = xy : \mathbb{R}^2 \longrightarrow \mathbb{R}^1$

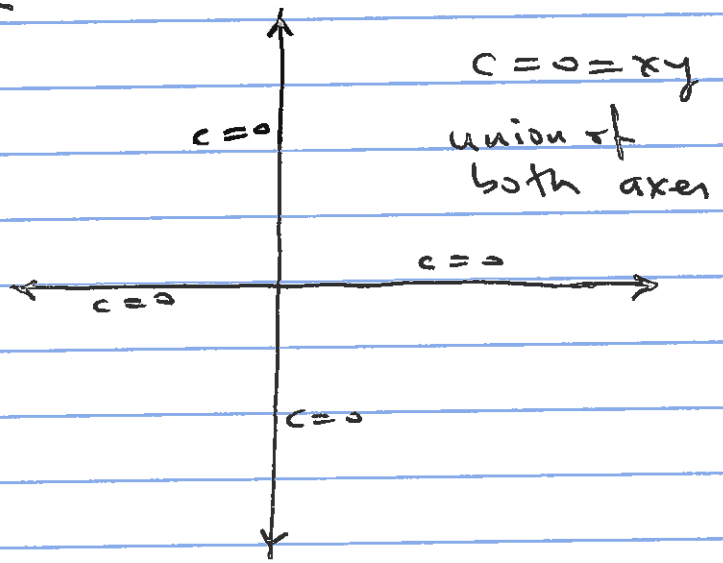
x, y $\quad$ z

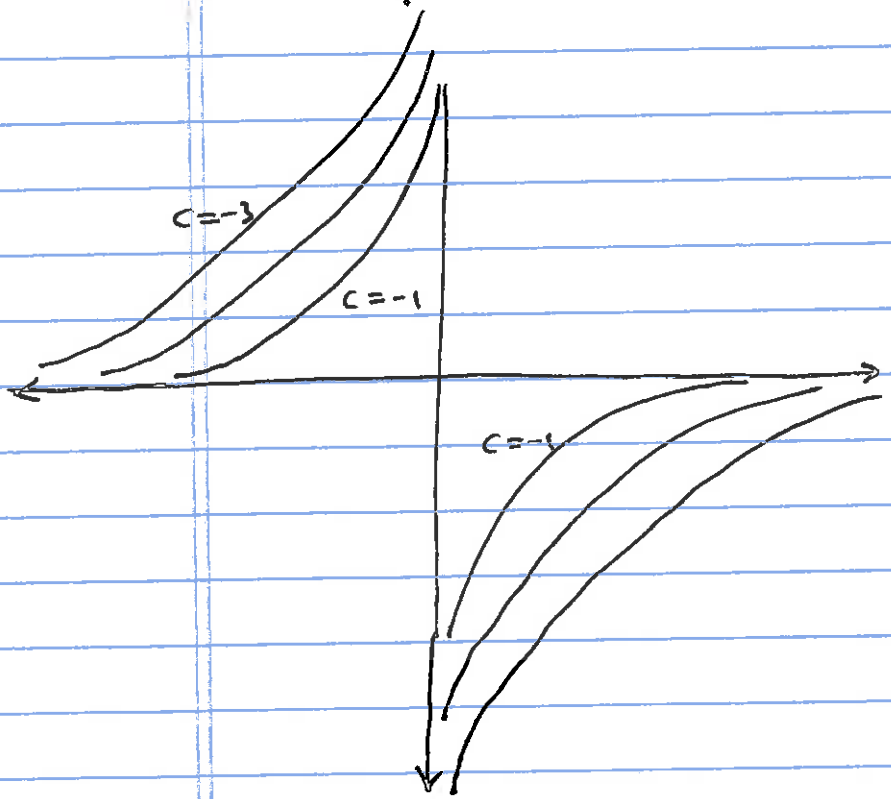
$z = xy$

Level sets $\{(x,y) \mid xy = c\}$

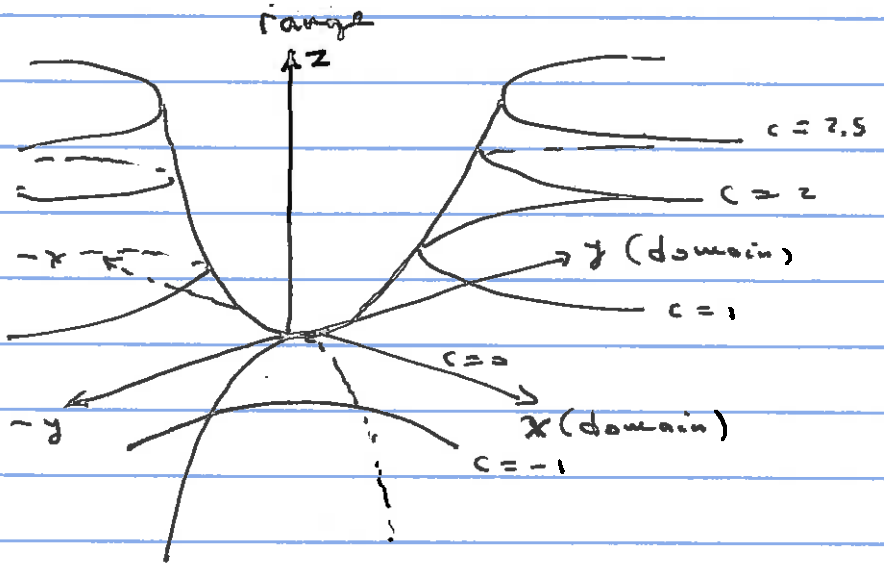


implicit graphs





Go to Next Page for graph (explicit)



$z = f(x,y)$

explicit graph.

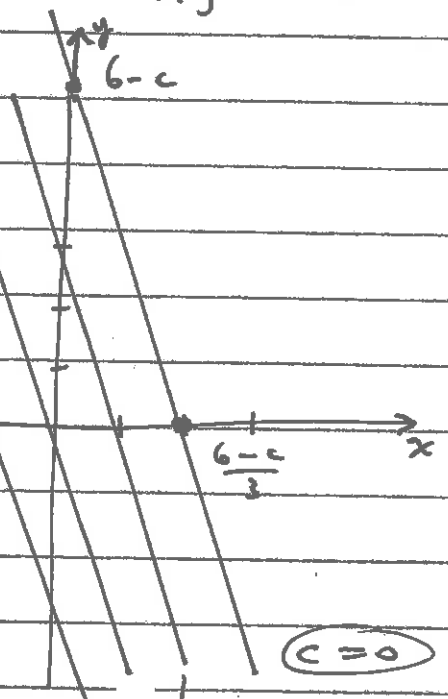
Example 3

$h(x,y) = 6 - 3x - y : \underset{x,y}{\mathbb{R}^2} \longrightarrow \underset{z}{\mathbb{R}}$

$h(x,y) = 6 - 3x - y = c$

$y = -3x + 6 - c$

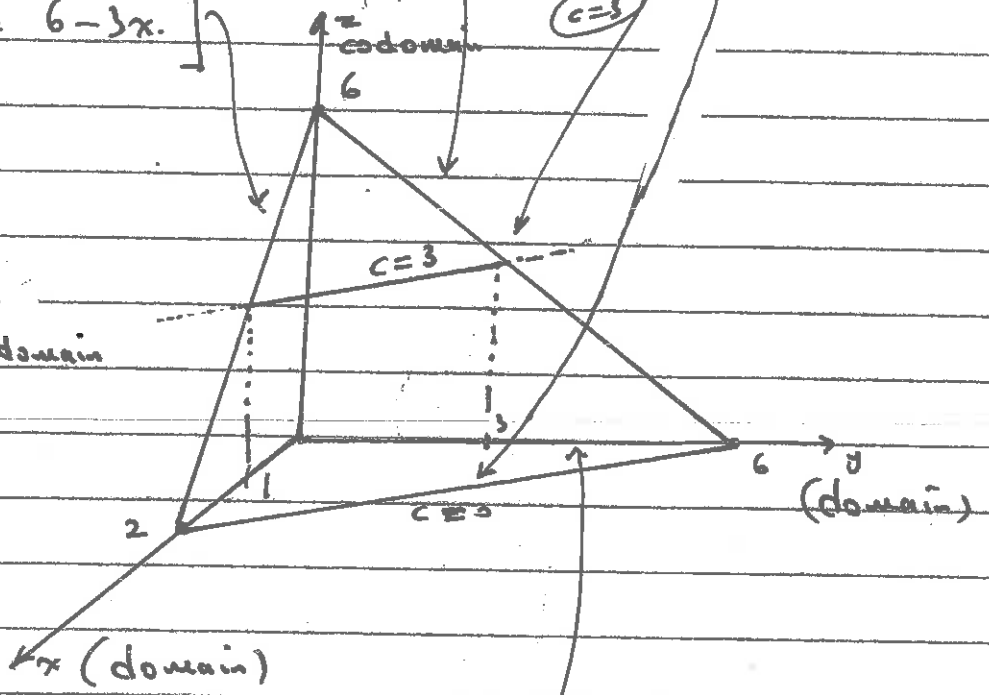
Level sets
Implicit graphs
in the Domain



Section

$x = 0$ (yz plane)
$z = h(0,y) = 6 - y$

$y = 0$ (xz plane)
$z = h(x,0) = 6 - 3x.$



(Explicit) Graph
$\subseteq$ Domain x Codomain

This is the plane

$z = 6 - 3x - y$

OR

$3x + y + z = 6.$

Ex 4

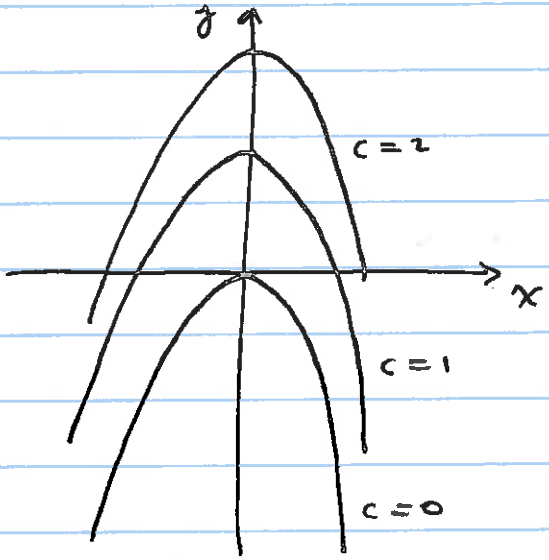
$$f(x,y) = y + x^2$$

(i) $z = c = f(x,y) = y + x^2$

$y = c - x^2$

} Horizontal slices by $z = c$ planes.

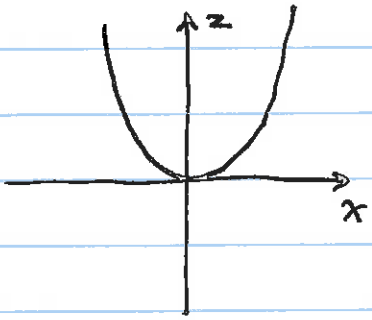
Contour curves / Implicit graph

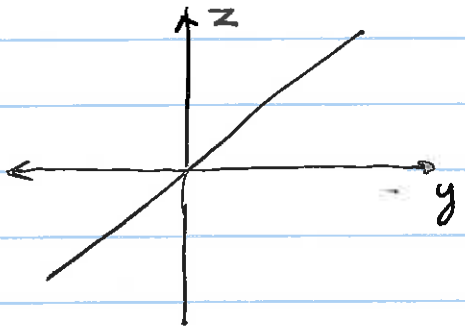


ii) $z = f(0,y) = y$

$z = f(x,0) = x^2$

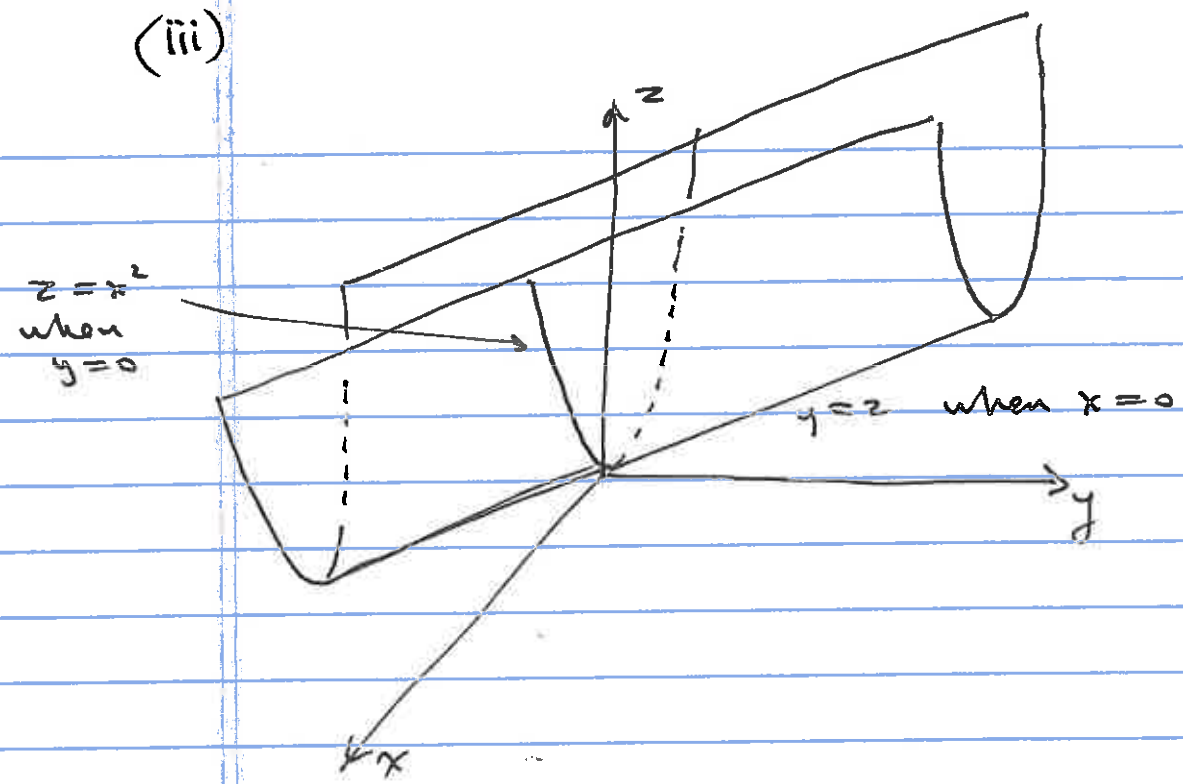
} vertical slices





(ii) Next page for the whole graph

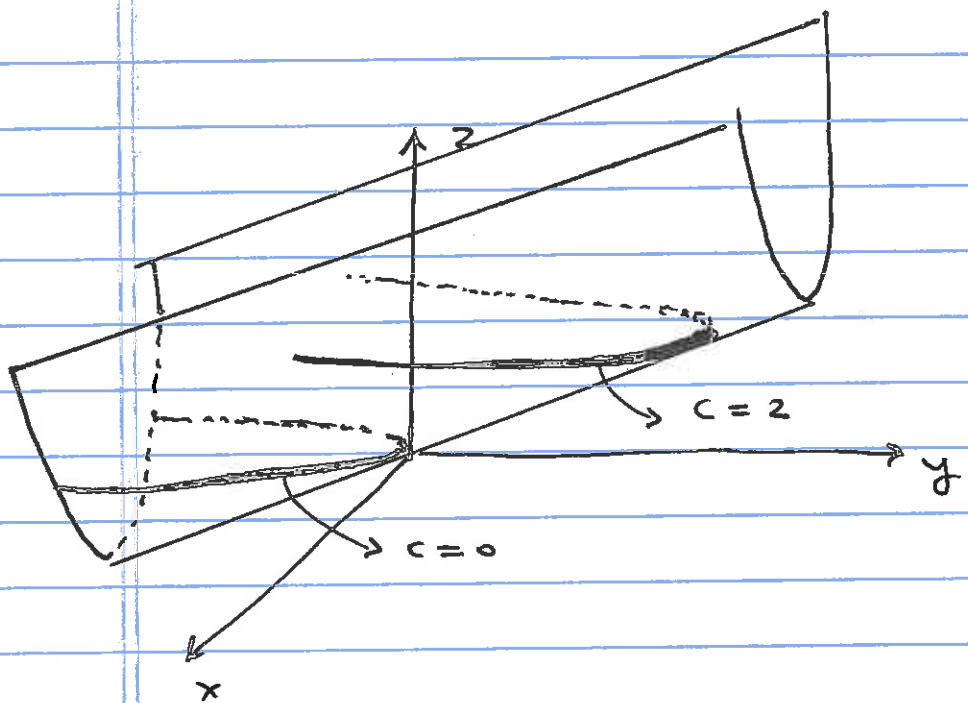
(iii)



Where are the horizontal slices?

(by $z = c$ planes)

Compare to (i)



Also look at the attached computer generated graphs.