2.1 Continue

GRAPHS:

Given $f: \underline{X} \subseteq \mathbb{R}^n \longrightarrow \underline{Y} \subseteq \mathbb{R}^m$

n = # of variables

m = # components of the function

- (Explicit) Graph of f

$$= \left\{ (\vec{x}, \overrightarrow{f(x)}) \in \mathbb{R}^n \times \mathbb{R}^m = \mathbb{R}^{n+m} \;\middle|\; \vec{x} \in \underline{X} \right\}$$

Domain $\times$ Codomain $\subseteq \mathbb{R}^{n+m}$

- Parametric Graph of f

$$= \left\{ f(\vec{x}) \in \mathbb{R}^m \;\middle|\; x \in \underline{X} \right\} \subseteq \text{Codomain}$$

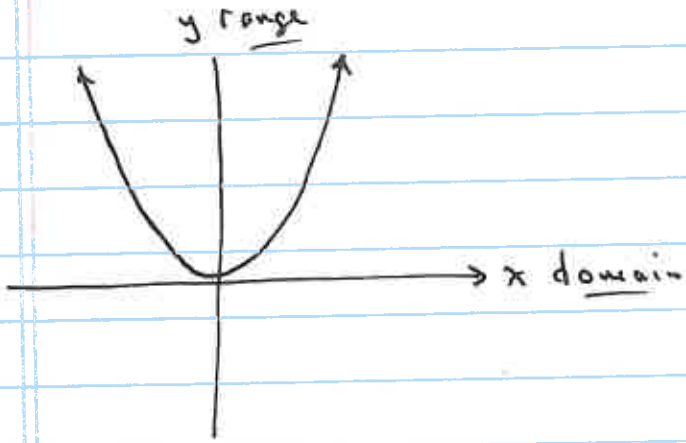
- Implicit graphs of f

$\forall b \in \underline{Y}$, the implicit graph corresponding to b is $\left\{ x \in \underline{X} \;\middle|\; f(x) = b \right\} \subseteq$ Domain

Also known as
- Level set,
- Contour graphs

Ex 1 $f(x) = x^2 : \mathbb{R}^1 \longrightarrow \mathbb{R}^1$

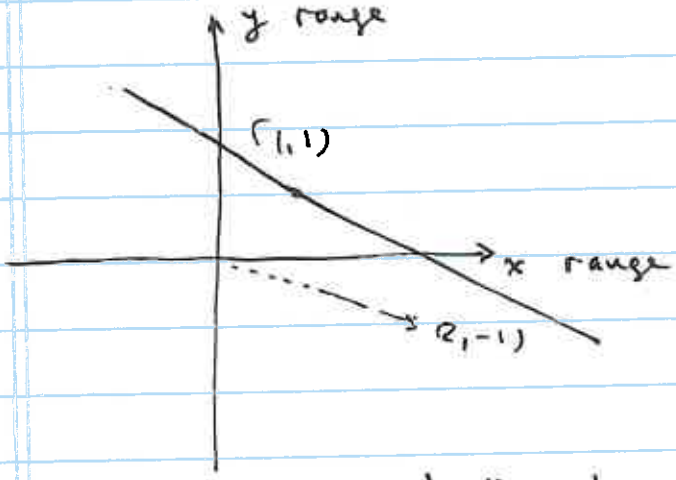
$x \qquad y$



(Explicit) graph

Ex 2 $g(t) = (1,1) + t(2,-1) : \mathbb{R}^1 \longrightarrow \mathbb{R}^2$

$t \qquad x, y$



parametric graph.

no mention of the domain variable t, it is supressed.

Example 3 $h(x,y) = x^2 + y^2 - 1$

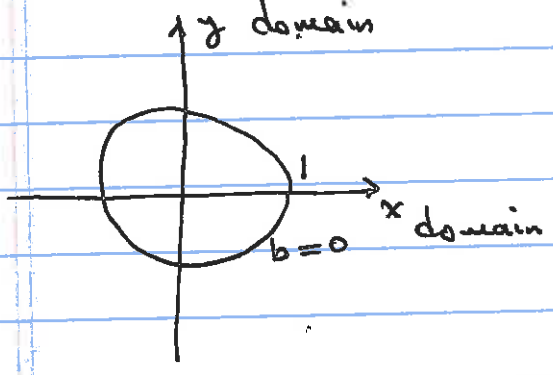
$h : \mathbb{R}^2 \longrightarrow \mathbb{R}$

$x, y \qquad z$

$b = 0 \qquad \{(x,y) \mid h(x,y) = 0\}$

$= \{(x,y) \mid x^2 + y^2 - 1 = 0\}$

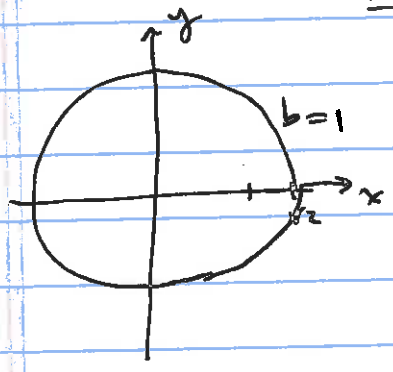
$= \{(x,y) \mid x^2 + y^2 = 1\} \subseteq$ Domain

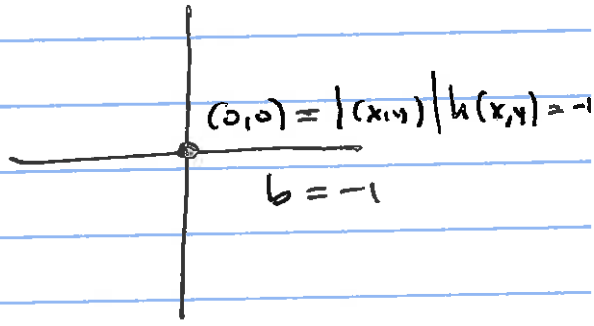


Implicit graphs

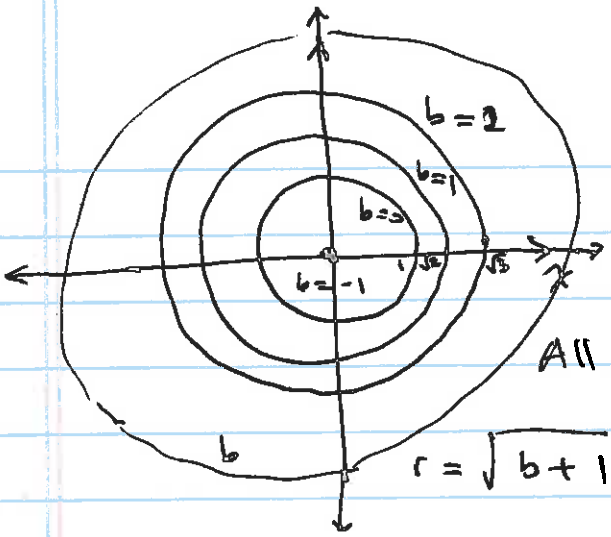
$b = 1 \qquad \{(x,y) \mid x^2 + y^2 - 1 = 1\}$

$= \{(x,y) \mid x^2 + y^2 = 2$





$b = -2 \quad : \quad \{(x,y) \mid x^2 + y^2 - 1 = -2\} = \emptyset.$



$\emptyset$ if $b < -1$.

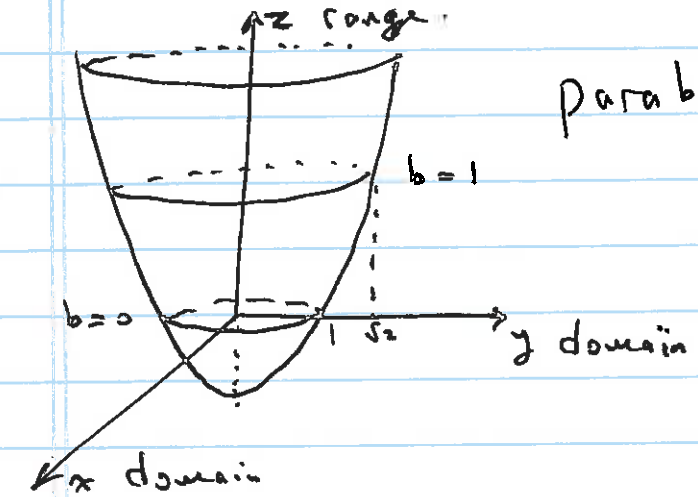
All circles with center $(0,0)$

if $b > -1$

Explicit graph of $h(x,y) = x^2+y^2-1$:

$h: \mathbb{R}^2 \longrightarrow \mathbb{R}$

x,y $\quad$ z



paraboloid

Ex 4 Topographical map of an island