① Feb 1, 2017

1.6 Matrix addition & Multiplication

Ex

$$2\begin{bmatrix} -1 & 6 & 0 \\ 5 & 7 & -1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 5 \end{bmatrix}$$

dimension of the matrix.

$\underset{\#\text{rows}}{2} \times \underset{\#\text{columns}}{3}$

$$= \begin{bmatrix} -2 & 12 & 0 \\ 10 & 14 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 8 \\ -4 & 8 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 12 & -8 \\ 14 & 6 & -22 \end{bmatrix}$$

- Multiplication

$\underset{m\times n}{A} \cdot \underset{p\times q}{B}$ exists (if and) only if $n = p$.

$\underset{m\times n}{A} \cdot \underset{n\times q}{B}$ is an $m\times q$ matrix

$\left.\begin{matrix} i^{th} \text{ Row} \\ j^{th} \text{ column} \end{matrix}\right\}$ entry of $A\cdot B$ is the dot product of i^{th} row of A and j^{th} column of B.

Ex ⓐ

$$\underset{2\times 3}{\begin{bmatrix} 2 & -1 & 5 \\ 3 & 4 & 0 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 3 & 1 \\ 5 & -1 \\ 2 & 6 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} \overbrace{2\cdot 3 - 1\cdot 5 + 5\cdot 2}^{11} & 33 \\ 29 & -1 \end{bmatrix}}$$

(b)

$$\underset{3\times 2}{\begin{bmatrix}3 & 1\\ 5 & -1\\ 2 & 6\end{bmatrix}}\underset{2\times 3}{\begin{bmatrix}2 & -1 & 5\\ 3 & 4 & 0\end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix}9 & 1 & 15\\ 7 & -9 & 25\\ 22 & 22 & 10\end{bmatrix}}$$

HW to read p 51-56.

CHAPTER II

2.1

Defn ① A function $f: X \to Y$ is a rule of assignment that associates to each element $x \in X$ a unique element $f(x) \in Y$.

X = Domain
Y = Codomain

$$\text{Range} = f(X) = \{ f(x) \mid x \in X \}$$

$$= \{ y \in Y \mid y = f(x) \text{ for some } x \in X \}$$

Defn ② (a) A function $f: X \to Y$ is called onto (surjective) if

$$f(X) = Y$$

($\Longleftrightarrow$ For each $b \in Y$, there is $a \in X$ s.t. $f(a) = b$)

$\forall$: for all
$\exists$: there exists

$$\forall b \in Y \; \exists a \in X \; \ni f(a) = b$$

(b) A function $f: X \to Y$ is called one-to-one (injective) if

$$\forall x_1, x_2 \in X, \; (f(x_1) = f(x_2) \implies x_1 = x_2)$$

or equivalently

$$\forall x_1, x_2 \in X \; (x_1 \neq x_2 \implies f(x_1) \neq f(x_2))$$

Contrapositives of each other

Example 1

$$f(x,y) = x^2 + y^2 - 4$$

$$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^1$$

f is not onto since there is no (x,y) s.t.

$$f(x,y) = -15,$$

Since $x^2+y^2-4 = -15$

$$x^2+y^2 = -11$$

but $x^2+y^2 \geq 0$.

Range of f = $[-4, \infty)$

f is not one-to-one because

$$f(2,1) = f(1,2) = 1 \quad \text{but} \quad (2,1) \neq (1,2)$$

Why?

one-to-one $\Longleftrightarrow \forall x_1, x_2 \ \left(f(x_1) = f(x_2) \implies x_1 = x_2 \right)$

not (one-to-one) $\Longleftrightarrow$ not $\left(\forall x_1, x_2 \ f(x_1) = f(x_2) \implies x_1 = x_2 \right)$

$\Longleftrightarrow \exists x_1, x_2 \quad f(x_1) = f(x_2)$ but $x_1 \neq x_2$

In Calculus III, we are interested in functions:

$$f : X \subseteq \mathbb{R}^n \longrightarrow Y \subseteq \mathbb{R}^m$$

$n = \#$ variables

$m = \#$ components.

Ex 2 $\quad g(x,y,z) = (\underbrace{x+y, \ x^2+y^3-z}_{2 \text{ components.}})$

$$\underset{3\text{ variables}}{g : \mathbb{R}^3} \longrightarrow \mathbb{R}^2$$

Is g 1-1 ? No :
$$\begin{cases} g(1,0,0) = (1,1) \\ g(0,1,0) = (1,1) \end{cases}$$

Is g onto ? YES $\quad \forall (a,b) \in \mathbb{R}^2$

$$g(0, a, a^3-b) = (a,b).$$

Ex $\quad f(t) = (3+t, 1-t, t)$:

$$f : \mathbb{R}^1 \longrightarrow \mathbb{R}^3$$

$\underline{f \text{ not onto}}$: No t exists s.t. $f(t) = (0,0,0)$

since $1-t=0 \Rightarrow t=1$

$t = 0 \Rightarrow t=0$

both can't happen.

PTO

$f(t) = (3+t, 1-t, t)$ is 1-1

Formal proof

$f(t_1) = f(t_2) \implies (3+t_1, 1-t_1, t_1) =$
$(3+t_2, 1-t_2, t_2)$

$\implies t_1 = t_2$.

$\forall t_1, t_2 \; (f(t_1) = f(t_2) \implies t_1 = t_2)$

So f is 1-1.