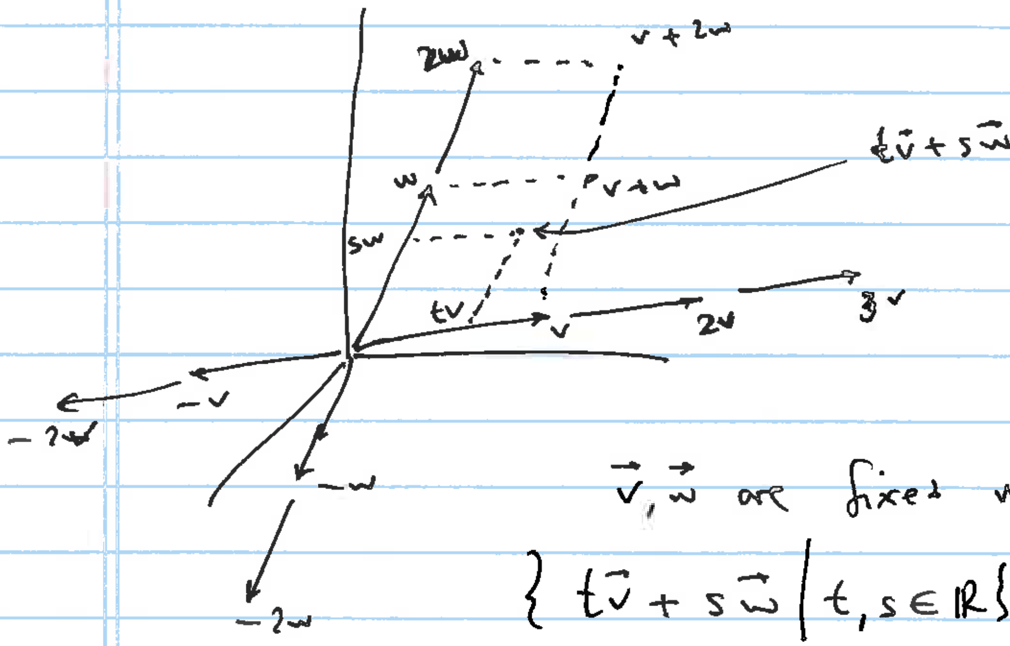
Jan 30, 2017

1

(1.5) Parametric Eqns of planes

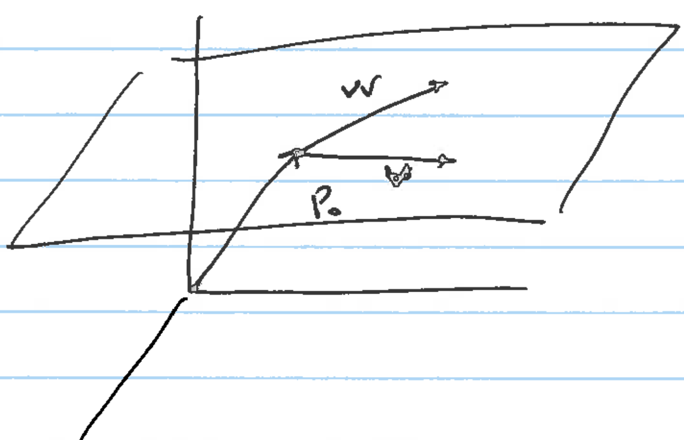


$\vec{v}, \vec{w}$ are fixed vectors, $\vec{v} \neq 0$, $\vec{w} \neq 0$, $v \not\parallel w$ not parallel

$$\underbrace{\{ t\vec{v} + s\vec{w} \mid t, s \in \mathbb{R} \}}$$

plane passing thru $\vec{0}$, parallel to $\vec{v}, \vec{w}$

This is "subspace generated by v & w" in Linear Algebra.



Affine plane

$\{ P_0 + t\vec{v} + s\vec{w} \mid t, s \in \mathbb{R} \}$ is a plane passing thru P_0 & parallel to $\vec{v}, \vec{w}$, provided that $v \neq 0$, $w \neq 0$, v not parallel to w

Exc #18 p 47. Find Parametric eq. for the plane passing thru $(2,9,-4)$ & parallel to $-8i+2j+5k$ & $3i-4j-2k$

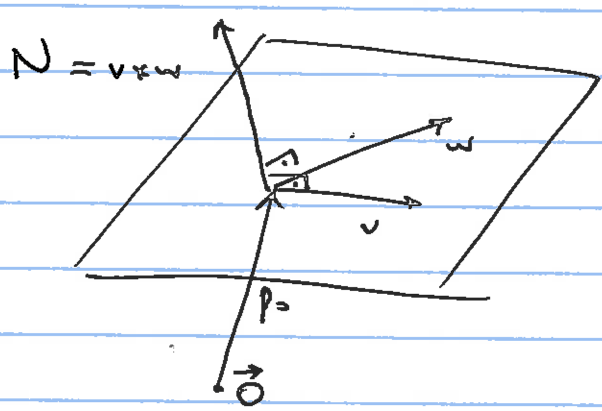
Ans. $r(s,t) = \underbrace{(2,9,-4) + t(-8,2,5) + s(3,-4,-2)}_{\text{vector parametric eq.}}$

OR

scalar parametric eqns

$$\begin{cases} x = 2 - 8t + 3s \\ y = 9 - 2t - 4s \\ z = -4 + 5t - 2s \end{cases}$$

(b) Find Closed Eqn for the same plane



$$(-8,2,5)\times(3,-4,-2) = \begin{vmatrix} i & j & k \\ -8 & 2 & 5 \\ 3 & -4 & -2 \end{vmatrix}$$

$N = v \times w = (16, -1, 26)$

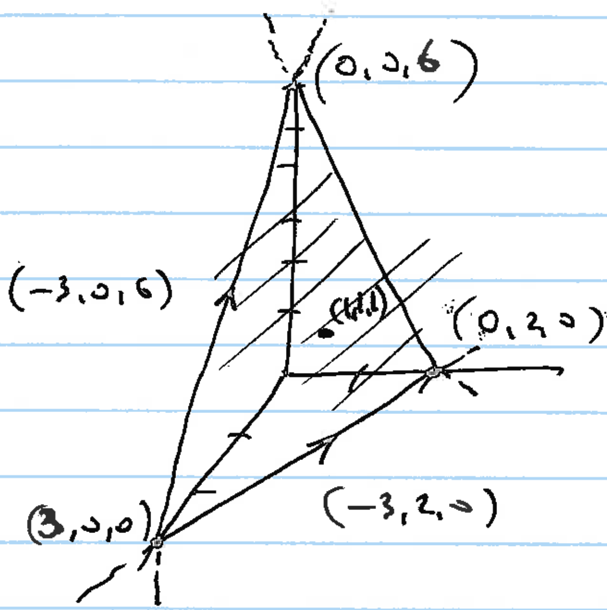
$$16x - y + 26z = 32 - 9 - 104 = -81$$

plug in $(2,9,-4)$

Ex) Given a closed eq^n.

$$2x + 3y + z = 6$$

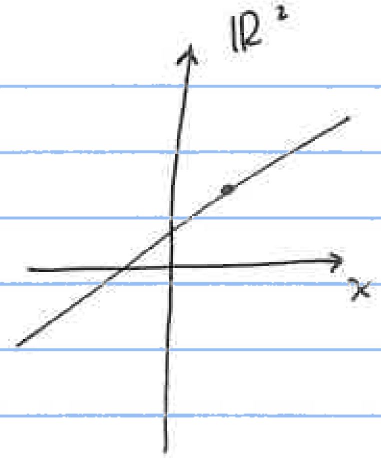
Plot the graph of the plane,
Find a parametric representation.



Parametric: $(3,0,0) + t(-3,2,0) + s(-3,0,6)$

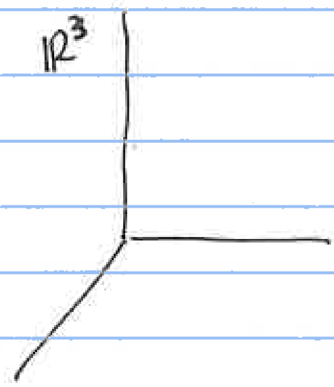
$t, s \in \mathbb{R}$

Parametric versus closed equations:



parametric line : $\vec{p}_0 + t\vec{v}_0$ 1-dimensional

One Closed eq. $Ax + By = C$ line 1-dimensional



2 equs. in $\mathbb{R}^3$

1-dimensional: line $p_0 + tv_0$	line: $\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3}$
2-dimensional plane $p_0 + t\vec{v}_0 + s\vec{w}_0$	$Ax + By + Cz = D$ plane

Informally:

① # of parameters $\geq$ dimension of graph

② (dim of Ambient Space) $-$ # equations $\leq$ dimension of graph

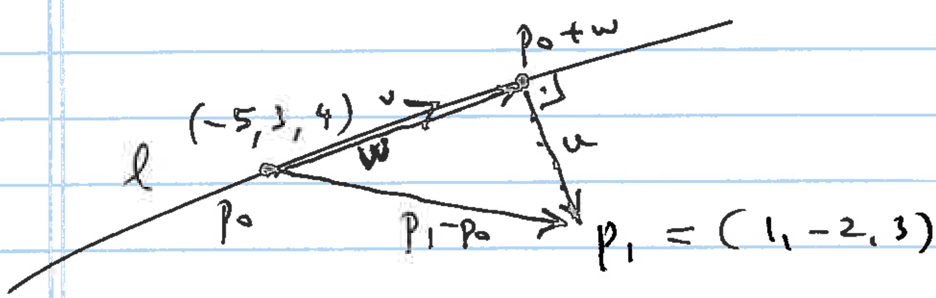
There are equalities in generic (maximum rank) cases.

Distance Problems.

Exc # 24 page 48

line ℓ: $\begin{cases} x = 2t - 5 \\ y = 3 - t \\ z = 4 \end{cases}$ $\qquad p_1 = (1, -2, 3)$

Find distance between p_1 & ℓ



$$p_1 - p_0 = (1, -2, 3) - (-5, 3, 4)$$
$$= (6, -5, -1)$$

$$r(t) = (-5, 3, 4) + t\underbrace{(2, -1, 0)}_{v}$$

$$w = \text{proj}_v (p_1 - p_0)$$

$$u = p_1 - (p_0 + w) = (p_1 - p_0) + w$$

$\|u\|$ = distance between p_1 & ℓ.

$$w = \text{proj}_{(2,-1,0)} (6,-5,-1)$$

$$= \frac{(6,-5,-1)\cdot(2,-1,0)}{(2,-1,0)\cdot(2,-1,0)} (2,-1,0)$$

$$= \frac{17}{5}(2,-1,0)$$

$$u = p_1 - (p_0 + w) = (1,-2,3) - \left((-5,3,4) + \frac{17}{5}(2,-1,0)\right)$$

$$= (p_1 - p_0) - w = (6,-5,-1) - \frac{17}{5}(2,-1,0)$$

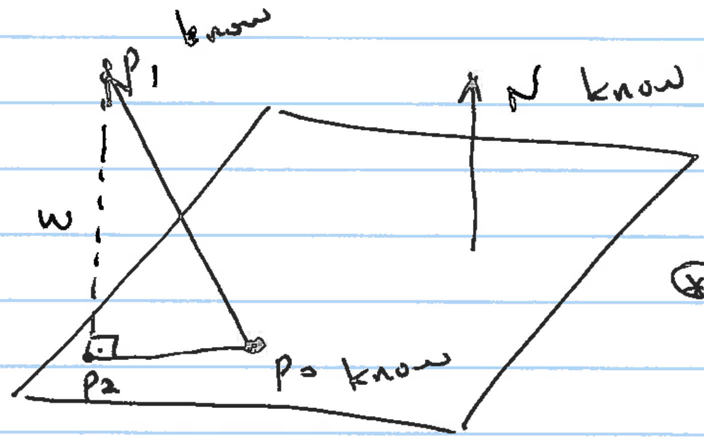
$$= \left(6 - \frac{34}{5}, -5 + \frac{17}{5}, -1\right)$$

$$u = \left(-\frac{4}{5}, \frac{-8}{5}, -1\right)$$

$$\text{dist} = \|u\| = \sqrt{\frac{16}{25} + \frac{64}{25} + 1} = \sqrt{\frac{105}{25}} = \frac{\sqrt{105}}{5}.$$

Exc.
p48 # 37

Ex) Distance from a pt p_1 to a plane given by a closed eqn.



$$(*) \quad \underbrace{N \cdot (\vec{x} - p_0) = 0}_{\text{plane}}$$

$$w = proj_N (p_1 - p_0)$$

$\|w\|$ = distance between p_1 & plane (*)

Simplify

$$\|w\| = \|proj_N (p_1 - p_0)\| = \left\| \frac{N \cdot (p_1 - p_0)}{N \cdot N} N \right\|$$

$$= \frac{|N \cdot (p_1 - p_0)|}{\|N\|^2} \|N\| = \frac{|N \cdot (p_1 - p_0)|}{\|N\|}$$