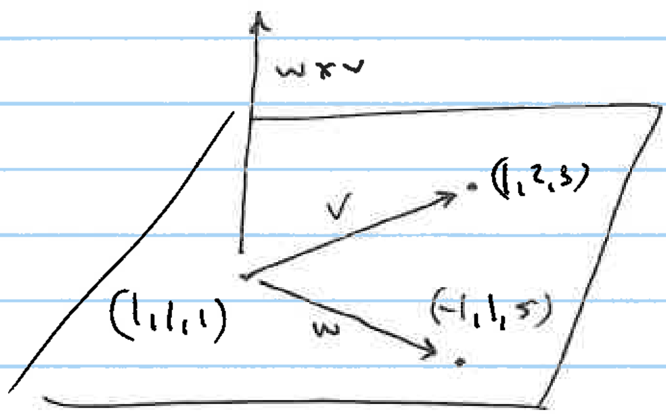
Jan 27, 2017

1

1.5 Continue

Ex $p_1 = (1,2,3)$
$p_2 = (1,1,1)$
$p_3 = (-1,1,5)$
Find an equation for the plane containing 3 pts.



$$v = (1,2,3) - (1,1,1) = (0,1,2)$$
$$w = (-1,1,5) - (1,1,1) = (-2,0,4)$$

$$w \times v = (-2,0,4) \times (0,1,2) = \begin{vmatrix} i & j & k \\ -2 & 0 & 4 \\ 0 & 1 & 2 \end{vmatrix}$$

$$= (-4, 4, -2)$$

Eqn of the plane:
$$-4x + 4y - 2z = (-4,4,-2)\cdot(1,2,3)$$
$$= -4 + 8 - 6$$

$$-4x + 4y - 2z = -2$$
OR $\boxed{2x - 2y + z = 1}$

Exc # 12 1.5 p 47 Find an equation of the plane containing ℓ_1 & ℓ_2

$$\ell_1 : \begin{cases} x = t + 2 \\ y = 3t - 5 \\ z = 5t + 1 \end{cases} \qquad \ell_2 : \begin{cases} x = 5 - t \\ y = 3t - 10 \\ z = 9 - 2t \end{cases}$$

To find a (all) pts of intersection

$$\begin{cases} t_1 + 2 = x = 5 - t_2 \\ 3t_1 - 5 = y = 3t_2 - 10 \\ 5t_1 + 1 = z = 9 - 2t_2 \end{cases}$$

① $t_1 + t_2 = 3$

② $3t_1 - 3t_2 = -5$

③ $5t_1 + 2t_2 = 8$

① $t_1 + t_2 = 3$

② $t_1 - t_2 = -\frac{5}{3}$

①+② $2t_1 = 3 - \frac{5}{3} = \frac{4}{3}$

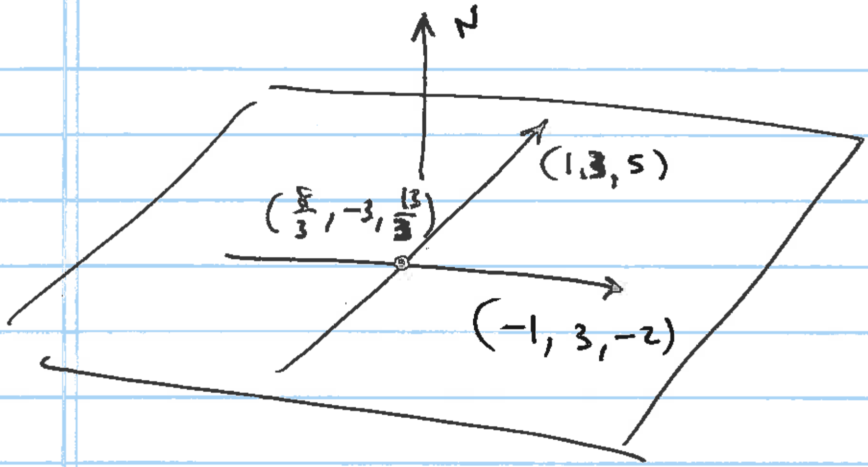
$t_1 = \frac{2}{3}$

$t_2 = \frac{7}{3}$

③ $5\left(\frac{2}{3}\right) + 2\left(\frac{7}{3}\right) = \frac{10+14}{3} = 8$ ✓

$$\begin{cases} x = \frac{2}{3} + 2 = \frac{8}{3} \\ y = 3\left(\frac{2}{3}\right) - 5 = -3 \\ z = 5\left(\frac{2}{3}\right) + 1 = \frac{13}{3} \end{cases}$$

only pt of intersection $\left(\frac{8}{3}, -3, \frac{13}{3}\right)$



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} + t_1 \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

$$N = (1,3,5) \times (-1,3,-2) = \begin{vmatrix} i & j & k \\ 1 & 3 & 5 \\ -1 & 3 & -2 \end{vmatrix}$$

$$= (-21, -3, 6)$$

Eq of the plane

$$(-21, -3, 6) \cdot \left[(x,y,z) - \left(\tfrac{8}{3}, -3, \tfrac{13}{3}\right) \right] = 0$$

$$-21x - 3y + 6z = (-21, -3, 6) \cdot \left(\tfrac{8}{3}, -3, \tfrac{13}{3}\right)$$

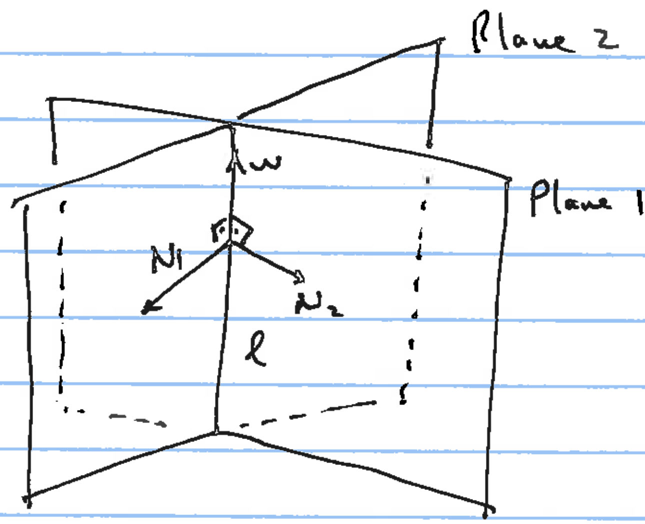
$$= -56 + 9 + 26$$

$$\boxed{-21x - 3y + 6z = -21}$$

Ex Find a parametric Eq for the line ℓ of intersection of two planes

Plane 1 $\quad x + 2y + 3z = 4$

Plane 2 $\quad 2x + 6y + 10z = 2$



$N_1 = (1, 2, 3) \perp$ Plane 1

$N_2 = (2, 6, 10) \perp$ Plane 2

$N_1 \perp \ell$

$N_2 \perp \ell$

$$\underbrace{N_1 \times N_2}_{\text{parallel to } \ell} = (1, 2, 3) \times (2, 6, 10) = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 6 & 10 \end{vmatrix}$$

$$= (2, -4, 2)$$

Eq of the line (parametric)

$$r(t) = \underbrace{(10, -3, 0)}_{\text{need a pt}} + t(2, -4, 2).$$

Try $z = 0$ $\quad x + 2y = 4$ $\quad$ any pt!

$2x + 6y = 2$

$x + 3y = 1$

$y = -3$

$x = 10$

$(10, -3, 0)$

Method II

Another Solution: Row Reduction:

$$x + 2y + 3z = 4$$
$$2x + 6y + 10z = 2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 6 & 10 & 2 \end{array}\right] \xrightarrow[-2R_1 + R_2]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & -6 \end{array}\right]$$

$$\xrightarrow[R_1 - R_2]{} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 10 \\ 0 & 2 & 4 & -6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 10 \\ 0 & 1 & 2 & -3 \end{array}\right]$$

parameter

$$x = 10 + t$$
$$y = -3 - 2t$$
$$z = t$$

$$(x, y, z) = (10, -3, 0) + t(1, -2, 1)$$

Compare to method I.