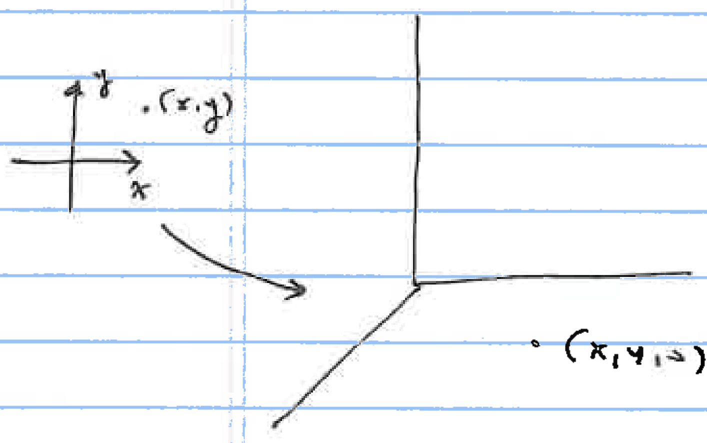
1.4 Continue

Does cross-product exist in $\mathbb{R}^2$?

$(a,b) \times (c,d)$?? Is it in $\mathbb{R}^2$?



Standard embedding of $\mathbb{R}^2$ into $\mathbb{R}^3$

$(x,y) \longmapsto (x,y,0)$

$$\circledast \quad (a,b,0) \times (c,d,0) = \begin{vmatrix} i & j & k \\ a & b & 0 \\ c & d & 0 \end{vmatrix}$$

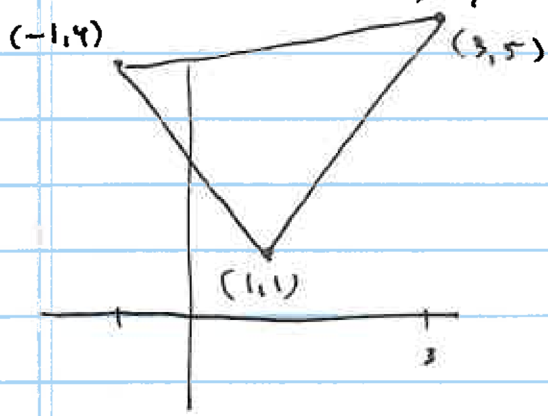
$$= (0, 0, ad-bc) \notin \mathbb{R}^2$$

So there is no cross product in $\mathbb{R}^2$, which produces a vector in $\mathbb{R}^2$.

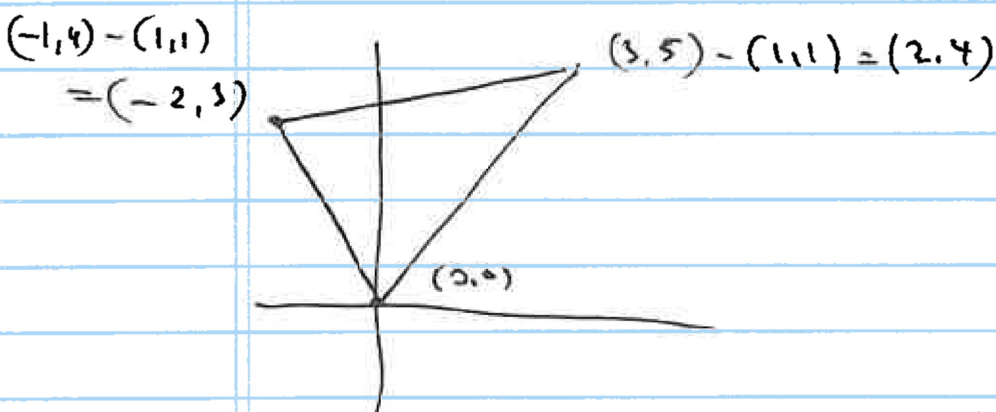
But, we still can use $\circledast$ to solve problems.

Ex) Find the area of the triangle in $\mathbb{R}^2$ with vertices

$$(1,1),\quad (3,5),\quad (-1,4).$$



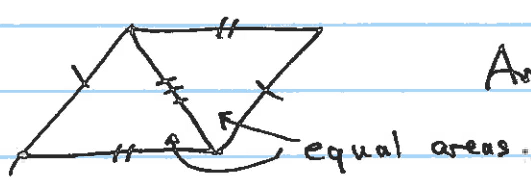
move the triangle in a parallel fashion so that $(1,1) \mapsto (0,0)$



$$(-2,3,0)\times(2,4,0) = \begin{vmatrix} i & j & k \\ -2 & 3 & 0 \\ 2 & 4 & 0 \end{vmatrix}$$

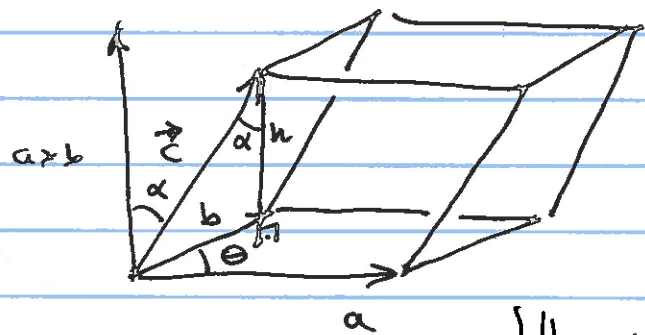
$$= (0,0,-14)$$

$14 = \|(0,0,-14)\|$ area of the parallogram.



Area of $\Delta = \dfrac{14}{2} = 7$.

Volume of a parallelopiped



6 faces
opposite faces are parallel to each other

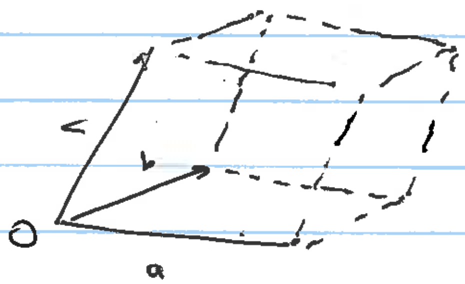
$h = \|c\| \cos\alpha$

$\Big\{ \|a \times b\| = \|a\|\|b\| \sin\theta$
area of the base

$$\text{Volume} = h \cdot \text{area of the base}$$
$$= \|c\| \cos\alpha \, \|a\| \, \|b\| \sin\theta$$
$$= \| (a \times b) \cdot c \|$$
$$= \left\| \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \right\|$$
$$= \left\| \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \right\|$$

Similar to Exc #18 p38

Volume of the parallelepiped



$a = 3i - j$

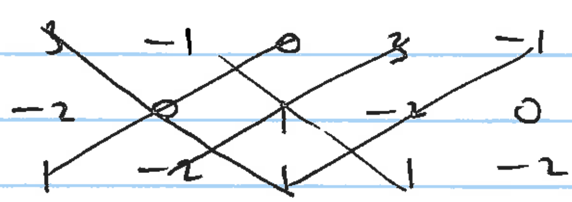
$b = -2i + k$

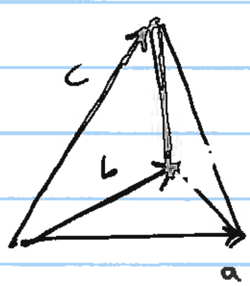
$c = i - 2j + k$

The vectors in the book are slightly different. This is a correct solution of what we started.

$$\text{Volume of P.P.} = \left| \begin{vmatrix} 3 & -1 & 0 \\ -2 & 0 & 1 \\ 1 & -2 & 1 \end{vmatrix} \right|$$

$$= \left| (0 - 1 + 0) - (0 - 6 + 2) \right| = \left| -1 + 4 \right| = 3$$





$$\text{Volume Tetrahedron} = \frac{1}{6} \text{ volume of parallelepiped} = \frac{1}{6} \cdot 3$$

$$= \frac{1}{2}.$$

A better picture.

1.5

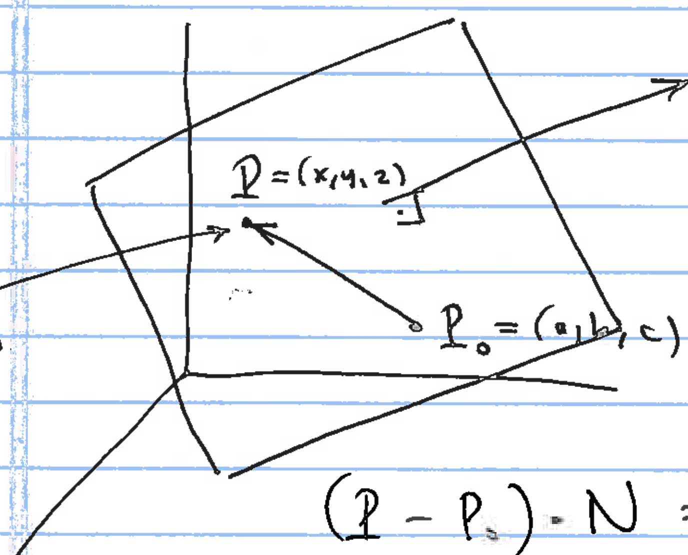
Lines : $\longrightarrow$ parametric $\quad r(t) = \vec{p}_0 + t\vec{v}$

$\longrightarrow$ symmetric $\quad \dfrac{x-a_1}{b_1} = \dfrac{y-a_2}{b_2} = \dfrac{z-a_3}{b_3}$

Planes : 1) closed

2) parametric

① Closed Equations of Planes



arbitrary pt.

$N = (A, B, C)$ Normal

A vector perpendicular to the plane.

a known pt of the plane

$$(P - P_0) \cdot N = 0$$

$$\big((x,y,z) - (a,b,c)\big) \cdot (A, B, C) = 0$$

$$(x-a, y-b, z-c) \cdot (A, B, C) = 0$$

$$A(x-a) + B(y-b) + C(z-c) = 0$$

$$Ax + By + Cz = D = Aa + Bb + Cc$$

1.5 Exc # 2 p 47

Find an equation of the plane containing $(9,5,-1)$ & $\perp$ $i - 2k$

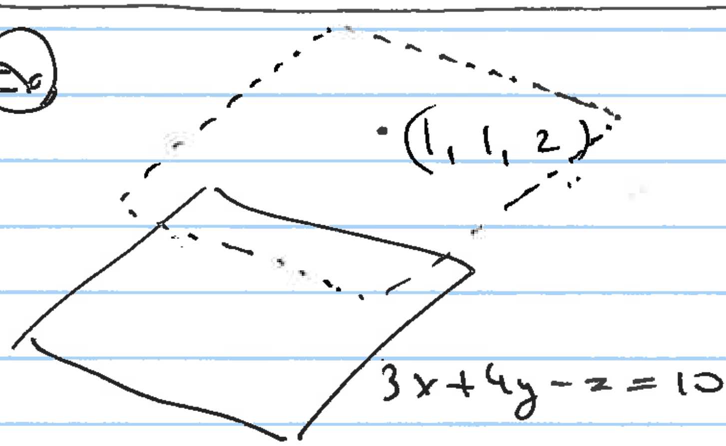
$$(x-9,\ y-5,\ z+1)\cdot(1,\ 0,\ -2) = 0$$

$$(x-9)\cdot 1 + (y-5)\cdot 0 + (z+1)(-2) = 0$$

$$x - 9 - 2z - 2 = 0$$

$$x - 2z = 11$$

Ex



Find an equation of the plane passing thru $(1,1,2)$ & parallel to the plane $3x + 4y - z = 10$

We can use same normals for parallel planes. $\rightarrow$ $N = (3,4,-1)$

Ans: $\underbrace{3x + 4y - z}_{(*)} = \underbrace{3\cdot 1 + 4\cdot 1 - 2}_{\text{plug in } (1,1,2) \text{ into } (*)} = 5$