1.2

In $\mathbb{R}^2$

$(*)$ $(a,b) = a\vec{i} + b\vec{j}$ $\qquad i=(1,0)$, $j=(0,1)$

Every vector in $\mathbb{R}^2$ can be represented uniquely as in $(*)$

In $\mathbb{R}^3$

$**$ $(a,b,c) = a\vec{i} + b\vec{j} + c\vec{k}$ $\qquad \begin{cases} i=(1,0,0) \\ j=(0,1,0) \\ k=(0,0,1) \end{cases}$

Every vector in $\mathbb{R}^3$ can be represented uniquely as in $**$

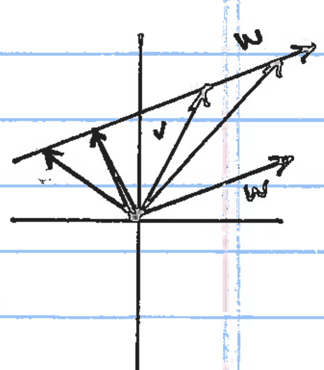
$\{i,j\}$ is a basis for $\mathbb{R}^2$

$\{i,j,k\}$ is a basis for $\mathbb{R}^3$

Caution 1) $(a_1, a_2, a_3, \dots a_n)$ $\begin{cases} \text{may mean a point} \\ \text{may mean a vector} \end{cases}$

2) $ai+bj+ck$ usually means a vector

Prop: $\{\vec{v}+t\vec{w} \mid t\in\mathbb{R}\}$ represents a line through $\vec{v}$, and parallel to w, provided that $\vec{w}\neq 0$.



Caution: the tips of the vectors.

Exc #14, p16

Line through $(12, -2, 0)$
$\parallel$ $5i - 12j + k$.

$$\underbrace{(x, y, z)}_{\text{dependent variables}} = r(t) = (12, -2, 0) + t(5, -12, 1)$$

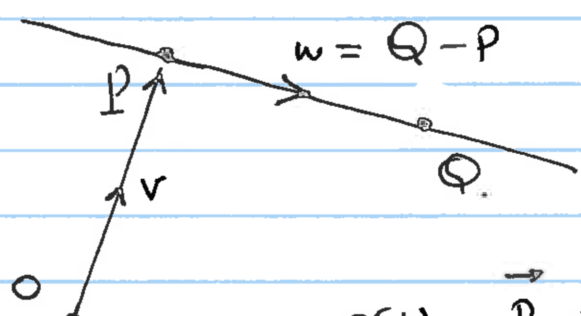
independent variable

$$= (12 + 5t, -2 - 12t, t)$$ vector parametric form.

OR one can also write:

$$\left.\begin{aligned} x &= 12 + 5t \\ y &= -2 - 12t \\ z &= t \end{aligned}\right\}$$ scalar parametric form.

Exc #16, p16 Line through $(2, 1, 2) = P$ and $(3, -1, 5) = Q$.



$$r(t) = P + t\vec{w}$$

$$r(t) = (2, 1, 2) + t[(3, -1, 5) - (2, 1, 2)]$$

$$= (2, 1, 2) + t[(1, -2, 3)]$$

$$= (2 + t, 1 - 2t, 2 + 3t)$$

Obs:

Line through P & Q

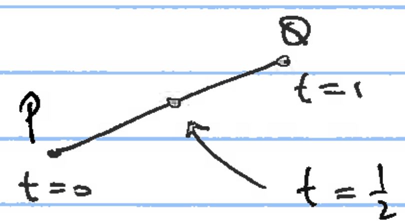
$$r(t) = P + t(Q - P)$$

$$= P + tQ - tP$$

Line: $r(t) = (1-t)P + tQ$ $\quad \underline{t \in \mathbb{R}}$

Question: What does "$r(t) = (1-t)P + tQ \quad 0 \le t \le 1$" represent?

Ans: Line segment from P to Q



$\frac{P+Q}{2}$. mid point of line segment.

Exc #28, p16 Are ℓ_1 & ℓ_2 same line?

*** use different t's for each line

ℓ_1: $x = 2t - 5$, $y = 3t + 2$, $z = 1 - 6t$

ℓ_2: $x = 1 - 2t'$, $y = 11 - 3t'$, $z = 6t' - 17$

$t = 1$ $\quad \left.\begin{array}{l} x = -3 \\ y = 5 \\ z = -5 \end{array}\right\} \ell_1$

$t' = 2$ $\quad \begin{array}{l} x = -3 \\ y = 5 \\ z = -5 \end{array}$

$t = 0$ $\quad \left.\begin{array}{l} x = -5 \\ y = 2 \\ z = 1 \end{array}\right\} \ell_1$

$t' = 3$ $\quad \begin{array}{l} x = -5 \\ y = 2 \\ z = 1 \end{array}$

Since ℓ_1 & ℓ_2 have 2 common distinct pts

$\ell_1 = \ell_2$, by "2 distinct points determine a line"

Exc # 42 p17

l_1: $x = 2t + 3$, $y = 3t + 3$, $z = 2t + 1$ $\qquad$ l_2: $x = 15 - 7t$, $y = t - 2$, $z = 3t - 7$

(*) Take different t's for each line

Need to find a common soln (if any) of

$$\begin{aligned} 2t_1 + 3 &= x \qquad & x &= 15 - 7t_2 \\ 3t_1 + 3 &= y & y &= t_2 - 2 \\ 2t_1 + 1 &= z & z &= 3t_2 - 7. \end{aligned}$$

$$\begin{aligned} 2t_1 + 3 &= 15 - 7t_2 \\ 3t_1 + 3 &= t_2 - 2 \\ 2t_1 + 1 &= 3t_2 - 7. \end{aligned}$$

$$\left.\begin{aligned} &(1) \quad 2t_1 + 7t_2 = 12 \\ &(2) \quad 3t_1 - t_2 = -5 \\ &(3) \quad 2t_1 - 3t_2 = -8 \end{aligned}\right\} \text{Does there exist any solution?}$$

Method I

(1), (3)

$$\begin{aligned} 2t_1 + 7t_2 &= 12 \\ -\ \ 2t_1 - 3t_2 &= -8 \\ \hline 10t_2 &= 20 \end{aligned}$$

$t_2 = 2$

$2t_1 + 14 = 12$

$2t_1 = -2$

$t_1 = -1$

must check #(2)

$3t_1 - t_2 \stackrel{?}{=} -5$

$-3 - 2 = -5$ ✓

consistent

$x = 2t_1 + 3 = 1$

$y = 3t_1 + 3 = 0$

$z = 2t_1 + 1 = -1$

pt of intersection

$(1, 0, -1)$

Method II Row Reduction

$$\begin{aligned} 2t_1 + 7t_2 &= 12 \\ 3t_1 - t_2 &= -5 \\ 2t_1 - 3t_2 &= -8 \end{aligned}$$

$$\left[\begin{array}{cc|c} 2 & 7 & 12 \\ 3 & -1 & -5 \\ 2 & -3 & -8 \end{array}\right] \xrightarrow[R_3 - R_1]{\text{Row Reduce}} \left[\begin{array}{cc|c} 2 & 7 & 12 \\ 3 & -1 & -5 \\ 0 & -10 & -20 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{10}R_3} \left[\begin{array}{cc|c} 2 & 7 & 12 \\ 3 & -1 & -5 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2 + R_3]{R_1 - 7R_3} \left[\begin{array}{cc|c} 2 & 0 & -2 \\ 3 & 0 & -3 \\ 0 & 1 & 2 \end{array}\right]$$

$$\xrightarrow[\frac{1}{3}R_2]{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2 - R_1]{} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

Consistent, unique sol^n.

$t_1 = -1$

$t_2 = 2$.

$$\begin{aligned} x &= 2t_1 + 3 = 1 \\ y &= 3t_1 + 3 = 0 \\ z &= 2t_1 + 1 = -1 \end{aligned} \qquad \begin{aligned} x &= 15 - 7t_2 = 1 \\ y &= t_2 - 2 = 0 \\ z &= 3t_2 - 7 = -1 \end{aligned}$$

$(1, 0, -1)$ pt of intersection.