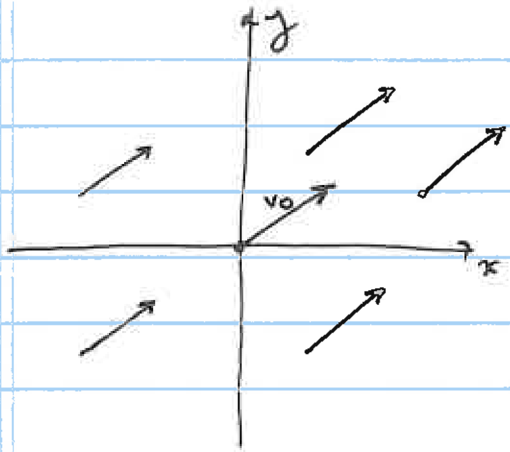
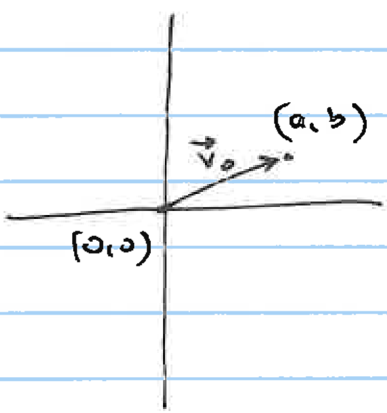
1.1 Continue

Vectors { · Direction
          · Magnitude



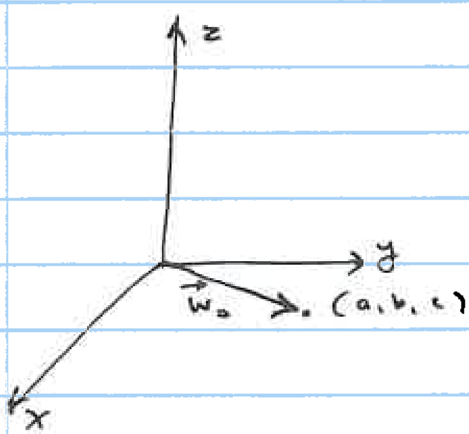
All have same direction
same magnitude

We take a canonical representative, the one starting at $\vec{0}$.



Associate (a,b) to this class of vectors

$$\vec{v}_0 = (a,b) = a\vec{i} + b\vec{j}$$



$$\vec{w}_0 = (a,b,c) = a\vec{i} + b\vec{j} + c\vec{k}$$

Algebraic Structure $\longleftrightarrow$ Geometric Structure

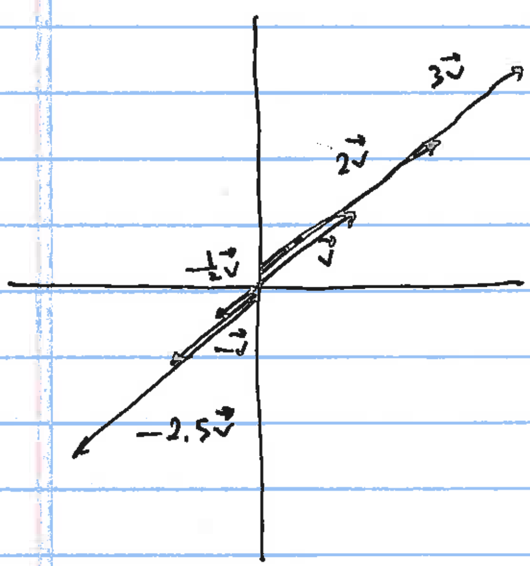
$+, \cdot$

- Lines through origin

If $\vec{v} \neq \vec{0}$ then

$\{ t \cdot \vec{v} \mid t \in \mathbb{R} \}$ represents the line through origin $\vec{0}$, and parallel to $\vec{v}$.

Caution: Only look at the collection of the tips of the vectors



Ex) Given $\vec{v} = (1, 3, -5)$:

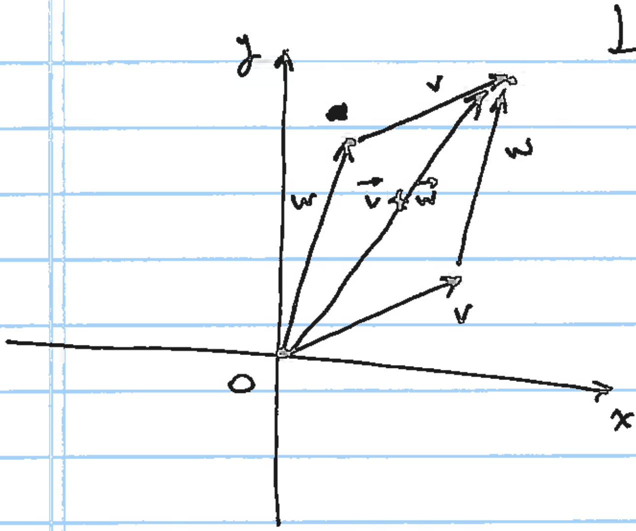
$\{ t(1,3,-5) \mid t \in \mathbb{R} \} = \{ (t, 3t, -5t) \mid t \in \mathbb{R} \}$

OR

$$\left. \begin{aligned} x &= t \\ y &= 3t \\ z &= -5t \end{aligned} \right\}$$ represents the line through $(0,0,0)$ & parallel to $(1,3,-5)$

1.1 Continue

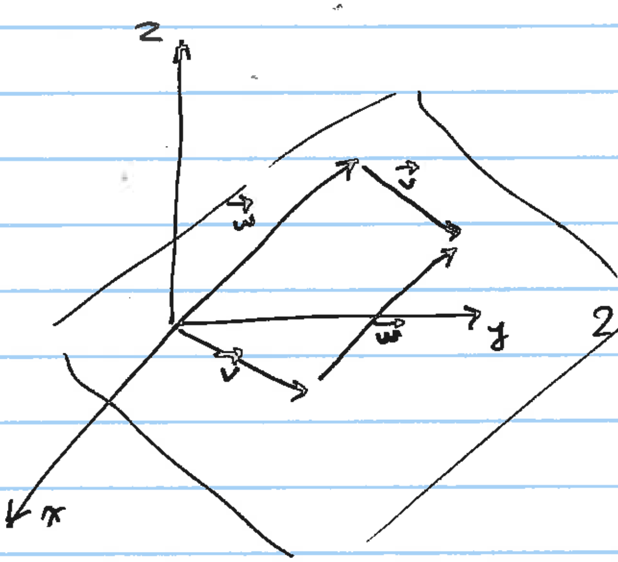
$\vec{v}+\vec{w}$ ----> what does it mean in geometric structure?



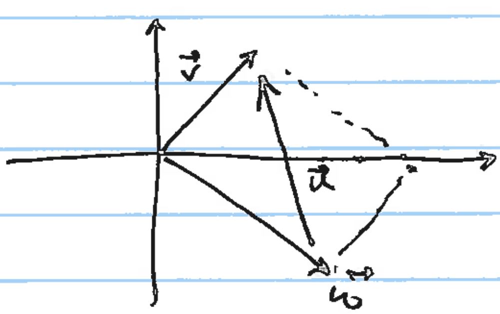
Let $\vec{v}=(a_1,b_1)$; $\vec{w}=(a_2,b_2)$

$$
\begin{aligned}
\vec{v}+\vec{w} &= (a_1,b_1)+(a_2,b_2)\\
&= (a_1+a_2,\ b_1+b_2)\\
&= (a_2+a_1,\ b_2+b_1)\\
&= (a_2,b_2)+(a_1,b_1)\\
&= \vec{w}+\vec{v}
\end{aligned}
$$

Head to Tail method OR Parallelogram Law.



There is a 2-dim'l plane containing the parallogram.

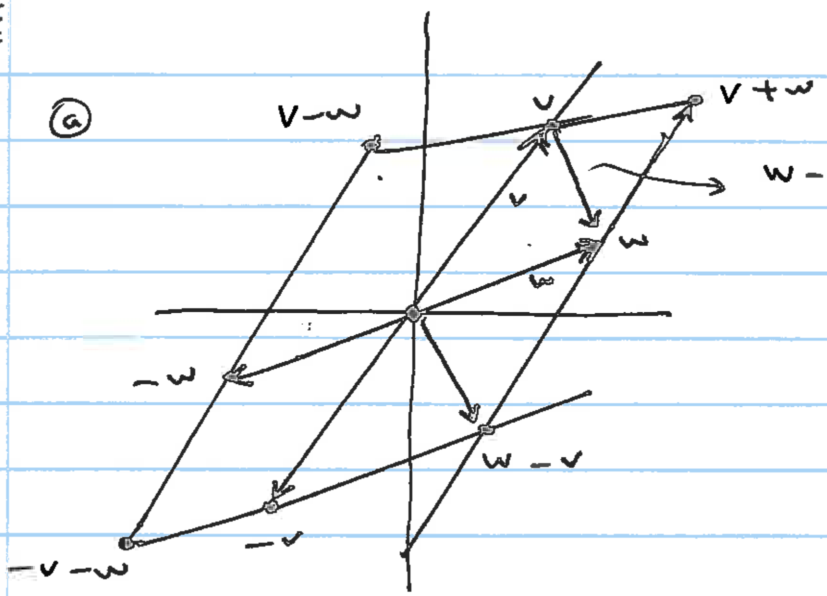


$\vec{w}+\vec{u}=\vec{v}$

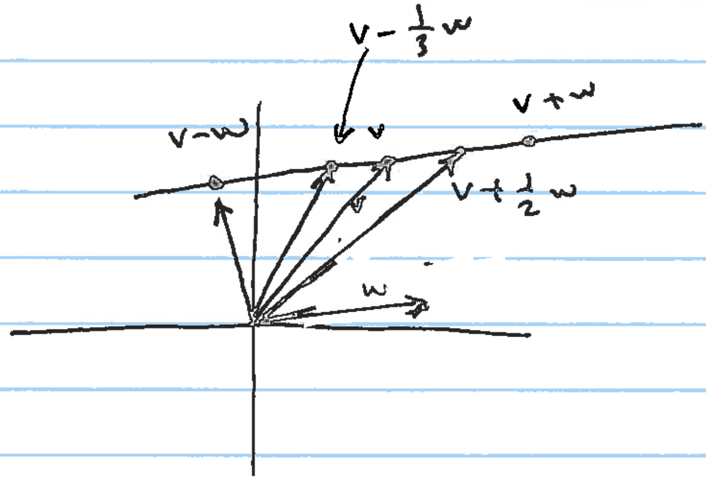
$\vec{u}=\vec{v}-\vec{w}$

Ex

(a)



(b)



Let v, w be s.t. $w \neq 0$

The tips of $\{ \vec{v} + t\vec{w} \mid t \in \mathbb{R} \}$ will lie on the line passing through v and parallel to $\vec{w}$.

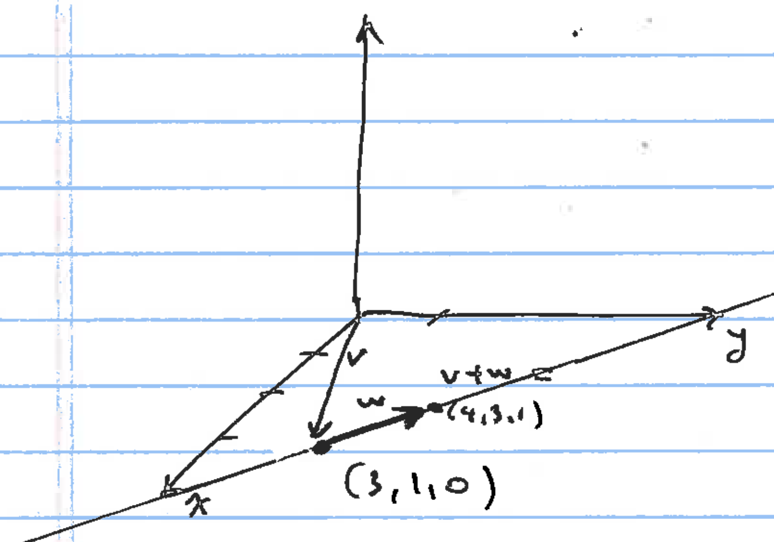
Ex

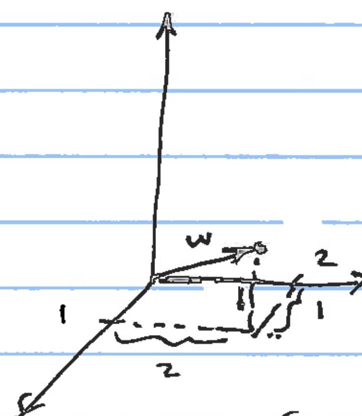
$$t \in \mathbb{R}: \quad \left.\begin{aligned} x &= 3 + t \\ y &= 1 + 2t \\ z &= 0 + t \end{aligned}\right\} \text{ is the line}$$

through $(3,1,0)$
& parallel to $(1,2,1)$

Why?

$$\begin{aligned}(x,y,z) &= (3+t,\ 1+2t,\ 0+t) \\ &= (3,1,0) + (t,2t,t) \\ &= (3,1,0) + t(1,2,1) \\ &= \vec{v} + t\vec{w}\end{aligned}$$





$w = (1,2,1)$

$(3,1,0) + (1,2,1) = (4,3,1)$

We'll do more of this in 1.2.

Defn Magnitude of a vector $\vec{v} = (a_1, a_2, \ldots a_n)$

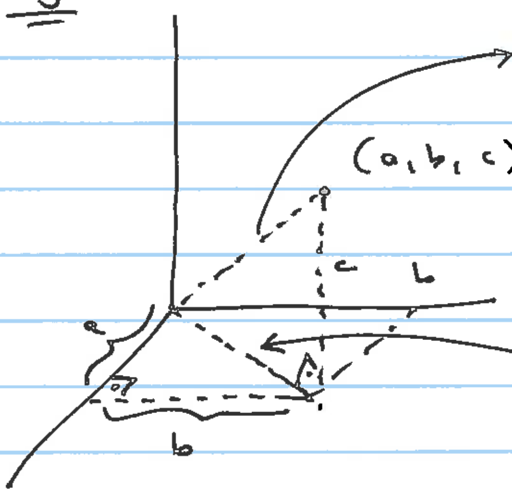
$$\|\vec{v}\| = \|(a_1, a_2, \ldots a_n)\| = \sqrt{a_1^2 + a_2^2 + \ldots + a_n^2}$$

Ex

$$\|(5, 1, 6)\| = \| 5\vec{i} + 1 \cdot \vec{j} + 6\vec{k} \|$$

$$= \sqrt{25 + 1 + 36} = \sqrt{62}.$$

Why?



$$\sqrt{c^2 + \left(\sqrt{a^2+b^2}\right)^2} = \sqrt{a^2+b^2+c^2}$$

$$\sqrt{a^2+b^2}$$