Jan 18, 2017

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1.1 Defn:

$$\mathbb{R}^n = \{ \underbrace{(a_1, a_2, \ldots, a_n) \mid a_i \in \mathbb{R} \text{ for } i = 1, 2, \ldots n}_{\text{The set of all ordered } n\text{-tuples of real \#s.}} \}$$

$\mathbb{R}$: Real numbers.

$\vec{a} = (a_1, a_2, \ldots a_n)$ book uses boldface

$\vec{x} = (x, y, z)$

not same.

Defn On $\mathbb{R}^n$ we have the following algebraic structure

$$+ : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

$$\cdot \quad \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

By:

$$(a_1, a_2, a_3, \ldots a_n) + (b_1, b_2, \ldots b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$$

$$k \cdot (a_1, a_2, a_3 \ldots a_n) = (ka_1, ka_2, \ldots, ka_n)$$

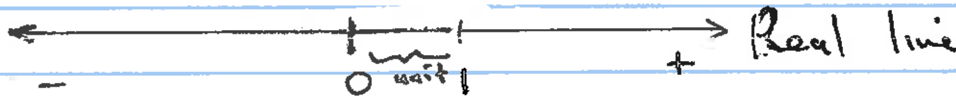
Properties: Commutativity, Associativity, etc.. p2-3
HW to read.

Algebraic Structure $\longleftrightarrow$ Geometric Structure

$(\mathbb{R}^n, +, \cdot)$ points, vectors, lines, planes, etc.

$\mathbb{R}^1$ $\xleftrightarrow[\text{correspondence}]{1-1}$ line

Real numbers



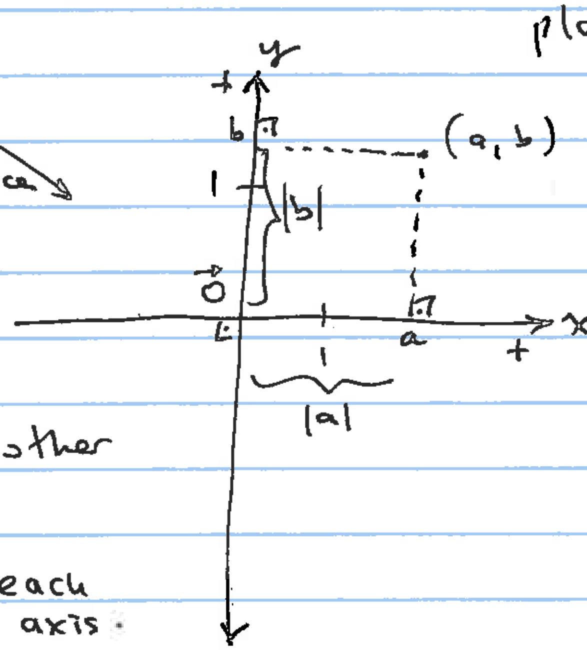
There is a one-to-one correspondence between real #'s and the points of a line where some choices are made:

- origin
- + - direction
- unit length

$\mathbb{R}^2 = \{(a,b) \mid a, b \in \mathbb{R}^2\}$

1 − 1 correspondence

plane



Fixed Choices:

Two axes, ⊥ each other

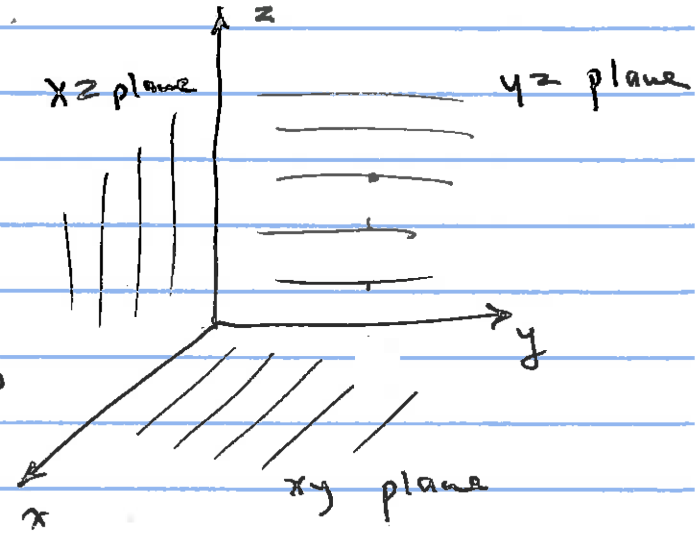
+ directions

unit lengths along each axis.

$\mathbb{R}^3$

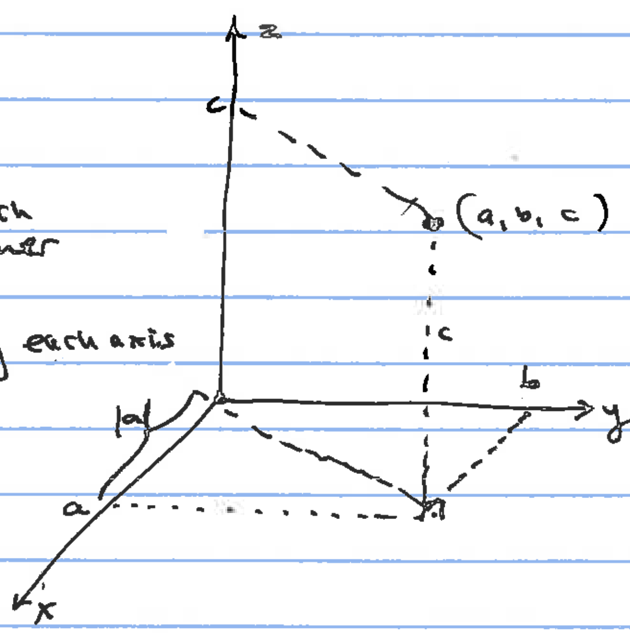
$\mathbb{R}^3 = \{\underbrace{(a,b,c)}_{\text{triples of real \#'s}} \mid a,b,c \in \mathbb{R}\}$

1 – 1 correspondence between $\mathbb{R}^3$ and the points of 3 space, once choices (*) below are made

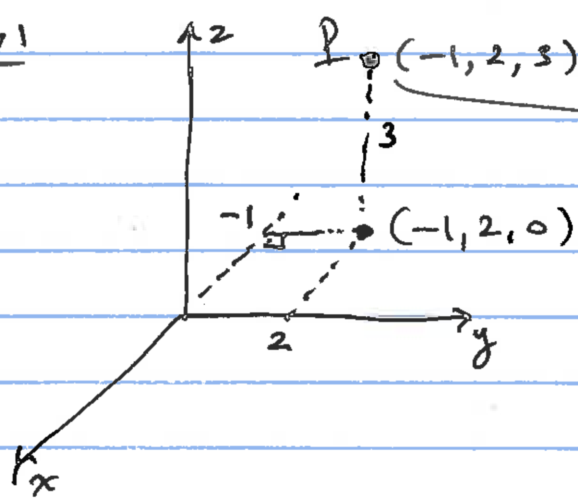


x, y, z axes.

(*)
- fixed x, y, z axes mutually ⊥ to each other
- unit lengths along each axis
- + directions for each axis.



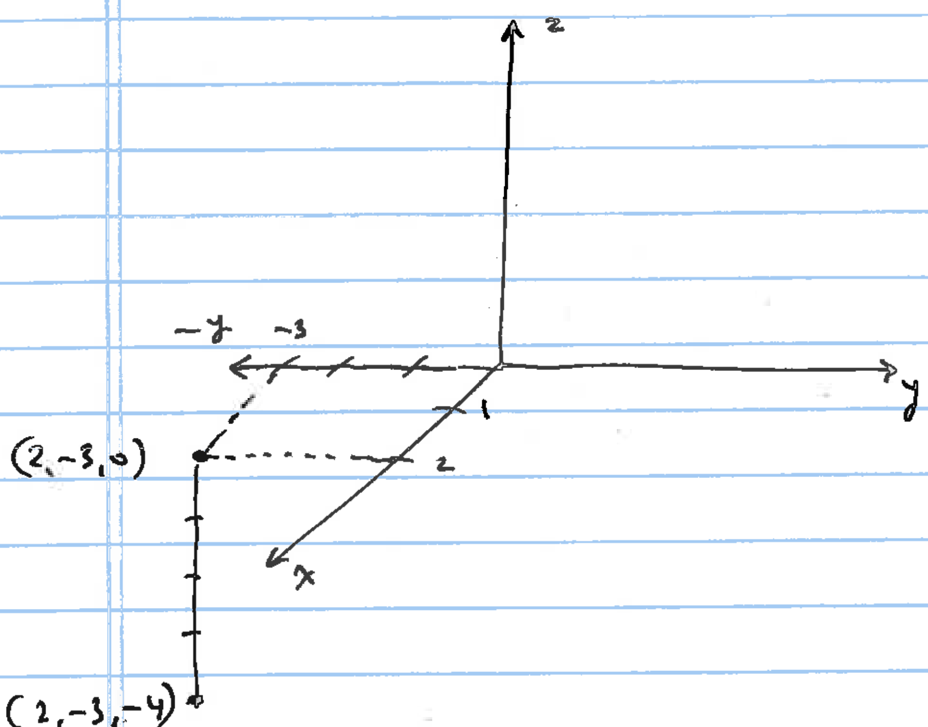
Ex 1



Q: What are the coordinates of the point P?

Ans. (−1, 2, 3)

Ex 2 Plot $(2, -3, -4)$



Ex 4b p 7

$$\frac{1}{2}(8,4,1) + 2\left(5,-7,\frac{1}{4}\right) = \left(4,2,\frac{1}{2}\right) + \left(10,-14,\frac{1}{2}\right)$$

$$= (14, -12, 1)$$

Ex

$$\underbrace{\overset{\text{in } \mathbb{R}^3}{(2,5,1)} + \overset{\text{in } \mathbb{R}^4}{(3,1,4,0)}}_{\text{can't add them}} =$$ This is not defined.

Ex Solve for x, y, z

$$(x,5,7) + (3,y,4) = (1,2,z)$$

$$(x+3,\ 5+y,\ 7+4) = (1,2,z)$$

$$\Rightarrow \begin{array}{l} x+3=1 \\ 5+y=2 \\ 7+4=z \end{array} ; \Rightarrow \begin{array}{l} x=-2 \\ y=-3 \\ z=11. \end{array}$$