DECEMBER 9

ANNOUNCEMENTS:

Final Exam

Dec 16, 2016 Friday

3:00 – 5:00 pm

66 SH (Same classroom as the lectures)

Office Hours Thursday 10:00 – 12:00
12/15/16 2:00 – 4:00

Review Session Wed Evening Time + Place TBA

Practice questions for Final are posted.

Please do complete ACE – Teaching Evaluations online through ICON

· Chapters 5, 6, 7 are in the final

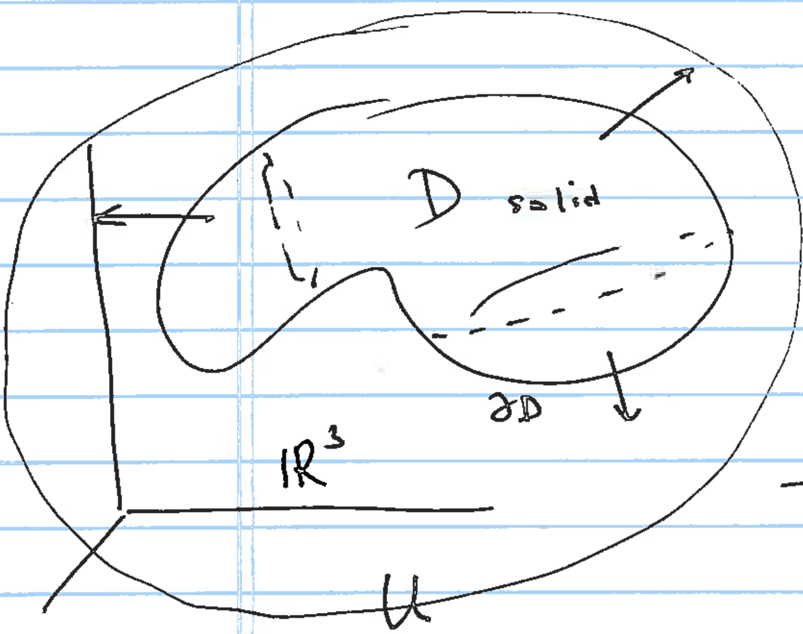
10 questions
~ 2 hrs.

Can bring one page of your own notes $8\frac{1}{2}'' \times 11''$.

7.3 Continue

THEOREM Divergence Thm / Gauss' Theorem

Let D be a bounded solid region in $\mathbb{R}^3$, whose boundary ∂D consists of finitely many piecewise-smooth closed orientable surfaces, oriented so that normals point outwards (away from D)



Let $F : U^{\text{open}} \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be a C^1 vector field

$D, \partial D \subseteq U \subseteq \mathbb{R}^3$

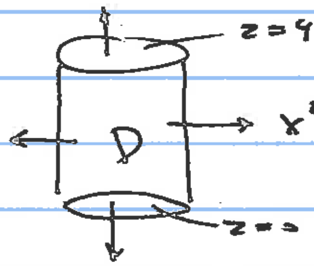
Then

$$\iiint_D \underbrace{\operatorname{div} F}_{\nabla \cdot F} \, dV = \iint_{\partial D} \vec{F} \cdot d\vec{S}$$

Example

Exc #14 (7.2) p489

Recall (p.3) On Dec 8 we calculated $\iint \vec{F}\cdot d\vec{S}$ Directly.



$D:\ 0 \le z \le 4$

$x^2+y^2 \le 9$

$\partial D = S$

$$\iint_{\partial D = S} (x,y,0)\cdot d\vec{S} \underset{\text{Gauss}}{=} \iiint_{D} \underbrace{\operatorname{div}(x,y,0)}_{2}\, dV =$$

$$\operatorname{div}(F_1,F_2,F_3) = \frac{\partial F_1}{\partial x_1} + \frac{\partial F_2}{\partial x_2} + \frac{\partial F_3}{\partial x_3}$$

$$\iiint_{D} 2\, dV = 2\cdot \underbrace{\text{volume}(D)}_{} = 2\cdot 36\pi = 72\pi$$

volume of solid cylinder $V = h\cdot \pi r^2$, $h = 4$, $r=3$

On Dec 8 $\iint_{S} F\cdot dS = 72\pi$ was calculated

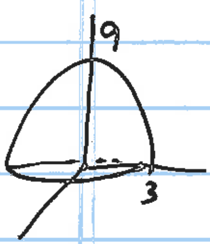
we see that they are equal.

p 505
Exc #6 (7.3)
Verify Gauss' Thm $\iiint_D \operatorname{div} F \, dV = \iint_{\partial D} \vec{F} \cdot d\vec{S}$
LHS RHS

$F = (x, y, z)$
$D = \{(x,y,z) \mid 0 \le z \le 9 - x^2 - y^2\}$

$\operatorname{div} F = 3$

LHS $\iiint_D \operatorname{div} F \, dV = \iiint_D 3 \, dV$



$$= \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} 3 \; dz\, dy\, dx$$

$$= \int_0^{2\pi} \int_0^3 \int_0^{9-r^2} 3\, r \; dz\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^3 3r(9-r^2)\, dr\, d\theta$$

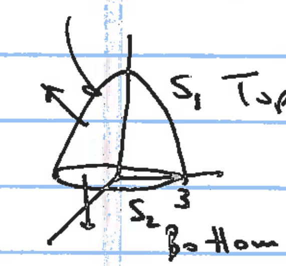
$$= \int_0^{2\pi} \int_0^3 (27r - 3r^3)\, dr\, d\theta$$

$$= 2\pi \cdot \left. \frac{27}{2} r^2 - \frac{3r^4}{4} \right|_0^3 = \frac{243\pi}{2}$$

RHS

$$\iint_{\partial D} (x,y,z)\cdot d\vec{S}$$

$z = 9 - x^2 - y^2$ $\qquad$ $\partial D = S_1 \cup S_2$



$\underline{X}(s,t) = (s\cos t, s\sin t, 9-s^2)$

$0 \le s \le 3$

$0 \le t \le 2\pi$

$\underline{X}_s = (\cos t, \sin t, -2s)$

$\underline{X}_t = (-s\sin t, s\cos t, 0)$

$N = (2s^2\cos t, 2s^2\sin t, s)$ upward normal

$$\iint_{S_1} F\cdot dS = \iint \underbrace{F(\underline{X}(s,t))}\cdot N(s,t)\,ds\,dt$$

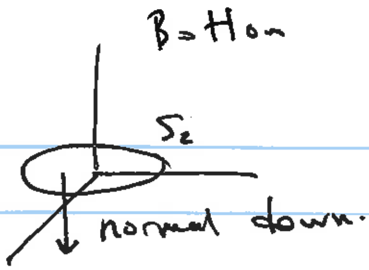
$$= \int_0^{2\pi}\int_0^3 (s\cos t, s\sin t, 9-s^2)\cdot(2s^2\cos t, 2s^2\sin t, s)\,ds\,dt$$

$$= \int_0^{2\pi}\int_0^3 \underbrace{2s^3\cos^2 t + 2s^3\sin^2 t}_{2s^3} + 9s - s^3\,ds\,dt$$

$$= \int_0^{2\pi}\int_0^3 9s + s^3\,ds\,dt$$

$$= 2\pi\cdot\left(\frac{9s^2}{2} + \frac{s^4}{4}\Big|_0^3\right) = \frac{243\pi}{2}$$

(5)



$$Y(s,t) = (s, t, 0) \qquad s^2+t^2 \le 9.$$

$$Y_s = (1,0,0)$$

$$Y_t = (0,1,0)$$

$$N_Y = (0,0,1) \qquad \text{normal} \uparrow$$

$$\iint_{S_2} F \cdot dS = - \iint_{s^2+t^2\le 9} F(Y(s,t)) \cdot N_Y(s,t)\, ds\, dt$$

$F(x,y,z) = (x,y,z)$

$$= - \iint \underbrace{(s,t,0)\cdot(0,0,1)}_{0}\, ds\, dt = 0$$

$$\underbrace{\frac{243\pi}{2}}_{\text{LHS}} = \iiint_D \operatorname{div} F \, dV = \iint_{\partial D} \vec{F} \cdot d\vec{S} = \iint_{\substack{S_1 \\ \text{Top}}} \vec{F} \cdot d\vec{S} + \iint_{\substack{S_2 \\ \text{bottom}}} F \cdot dS$$

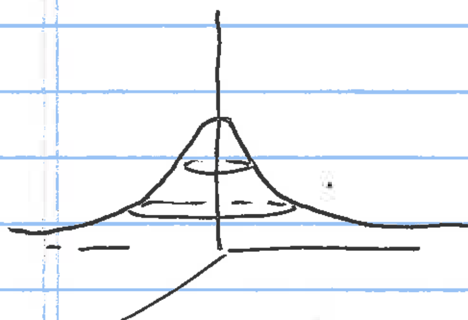
$$= \underbrace{\frac{243\pi}{2} + 0}_{\text{RHS}}$$

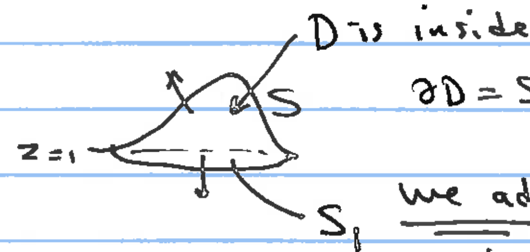
p 506

Exc #18 $\quad F = (x, y, 2-2z)$

$S =$ graph of $z = e^{1-x^2-y^2}$

above $z = 1$





$\partial D = S \cup S_1$

we add to close it up.

$$\underbrace{\iint_S \vec{F} \cdot d\vec{S}}_{\text{want}} + \iint_{S_1} \vec{F} \cdot d\vec{S} \overset{\text{Gauss'}}{=} \underbrace{\iiint_D \operatorname{div} F \cdot dV}_{0} = 0$$

$\operatorname{div} F = 0$

S_1: $\quad X(s,t) = (s, t, 1)$ $\quad$ ~~disk~~ circle on the plane $z=1$

needed normal $N = (0, 0, -1)$; $\quad N_X = (0, 0, 1)$

opposite normal

$$-\iint_{S_1} \vec{F} \cdot d\vec{S} = \iint \underbrace{\underbrace{F(\vec{X}(s,t))}_{(s,\,t,\,0)\cdot(0,\,0,\,-1)} \cdot \vec{N}_X}_{0} \cdot ds\,dt = 0$$

$$\iint_S \vec{F} \cdot d\vec{S} = 0$$