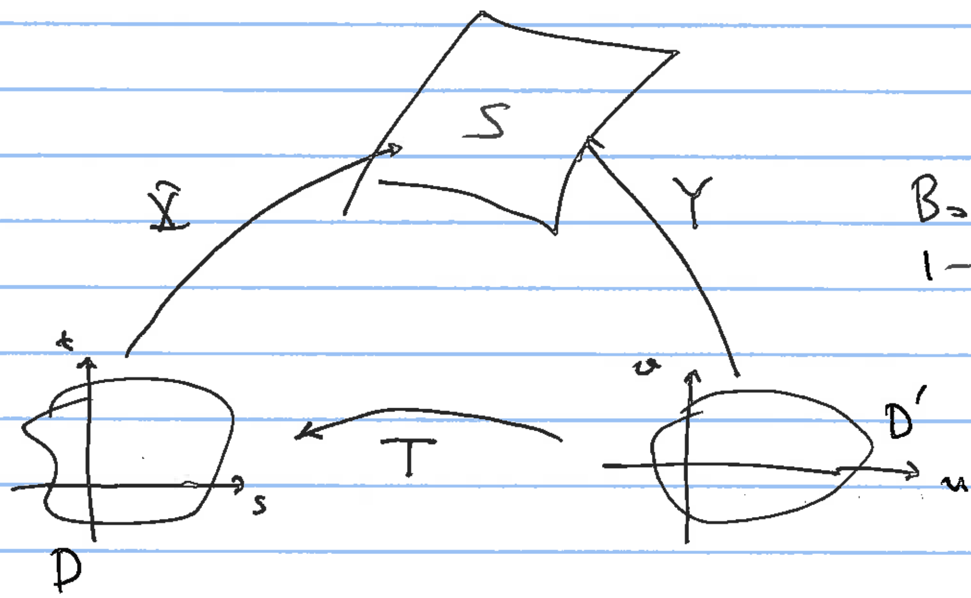
7.2 Finalize

Equivalent parametrizations



Both smooth 1-1 parametrizations

Coordinate Transformation $\quad T(u,v) = (s,t)$: T 1-1, onto, $\det DT \neq 0$

and $\left[\begin{array}{l}\det DT \text{ have same sign} \\ \text{on } D'\end{array}\right.$

$$\mathbf{X}(T(u,v)) = \mathbf{Y}(u,v)$$

$f: S \longrightarrow \mathbb{R}^1$ continuous

$F: S \longrightarrow \mathbb{R}^3$ continuous

$$\iint_{\mathbf{X}} f\, dS = \iint_{\mathbf{Y}} f\, dS$$

$$\iint_{\mathbf{X}} \vec{F} \cdot d\vec{S} = \pm \iint_{\mathbf{Y}} \vec{F} \cdot d\vec{S}$$

$+$ if $\det DT > 0$

$-$ if $\det DT < 0$

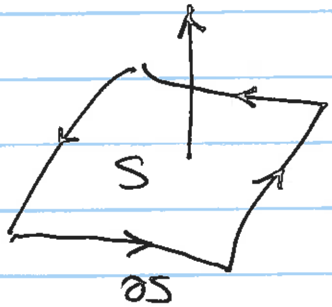
$\dfrac{\partial(s,t)}{\partial(u,v)}$

Reason :

$\vec{N}_{\mathbf{Y}} = \vec{N}_{\mathbf{X}} \cdot \det DT$

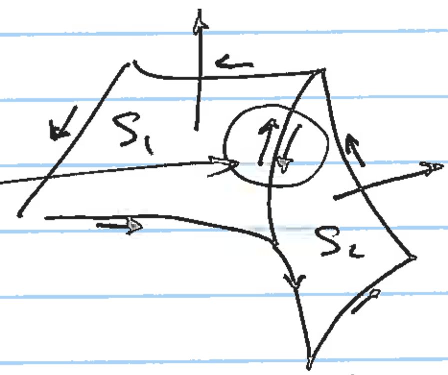
Recall orientation of a curve is a direction of the curve

Recall orientation of a surface is a choice of a side:



compatible orientation of S and ∂S

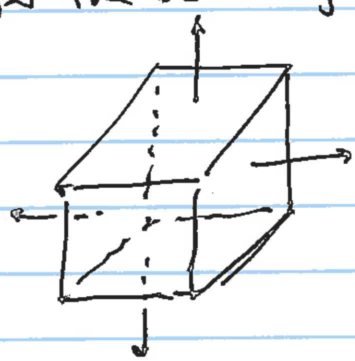
By Right Hand Rule



correctly attaching two oriented surface patches together

Once S_1 & S_2 are joined along the common edge, that is no longer the boundary of the union.

Ex.

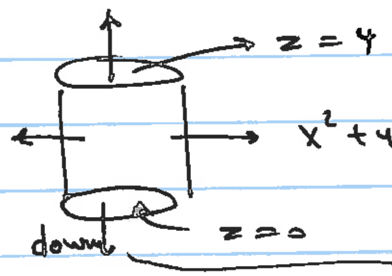


Surface of a cube.
piecewise smooth surface, Oriented.
All normals are outward.

Exc

7.2 #14 p 489 Closed cylinder, outward normal

S :



$$\iint_S (x\vec{i} + y\vec{j}) \cdot d\vec{S}$$

Top

For X, Y: D

$s^2+t^2 \le 9$

$X = (s, t, 4)$

$X_s = (1,0,0)$

$X_t = (0,1,0)$

$N_X = (0,0,1)$

correct normal/up.

Bottom

$Y = (s, t, 0)$

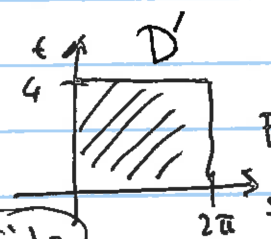
$Y_s = (1,0,0)$

$Y_t = (0,1,0)$

$N_Y = (0,0,1)$ up

wrong normal

Side



$Z = (3\cos s, 3\sin s, t)$

$Z_s = (-3\sin s, 3\cos s, 0)$

$Z_t = (0, 0, 1)$

$N_Z = (3\cos s, 3\sin s, 0)$

correct normal.

$$72\pi = \iint_S \vec{F}\cdot d\vec{S} = +\iint_X \vec{F}\cdot d\vec{S} - \iint_Y F\cdot dS + \iint_Z \vec{F}\cdot d\vec{S}$$

$$\iint_X F\cdot dS = \iint_{\substack{D' \\ s^2+t^2\le 9}} (x,y,0)\cdot d\vec{S} = \iint_{s^2+t^2\le 9} (s,t,0)\cdot(0,0,1)\,drdt = 0$$

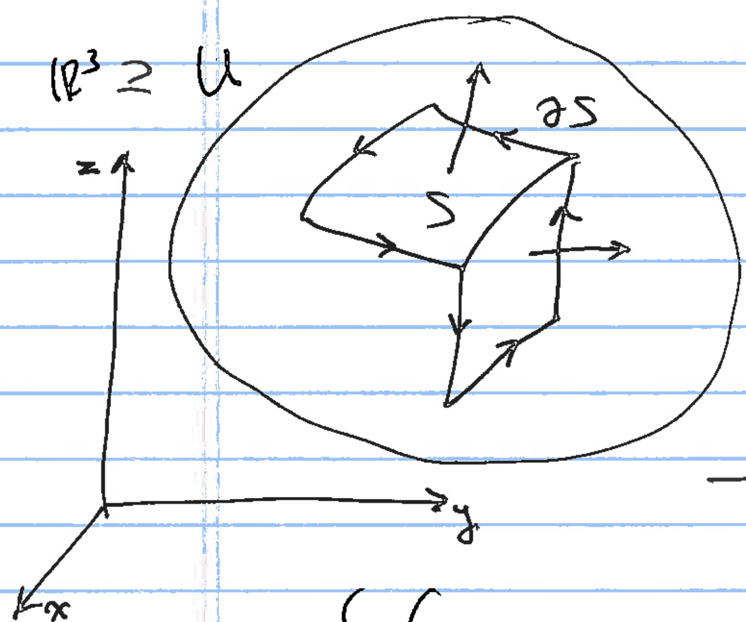
$$\iint_Y F\cdot dS = \iint_{\substack{D' \\ s^2+t^2\le 9}} (x,y,0)\cdot d\vec{S} = \iint_{s^2+t^2\le 9} (s,t,0)\cdot(0,0,1)\,drdt = 0$$

$$\iint_Z F\cdot dS = \underset{(D)}{\int_0^{2\pi}\int_0^4} \underbrace{\overbrace{(3\cos s,\ 3\sin s, 0)}^{F(Z(s,t))}\cdot\overbrace{(3\cos s, 3\sin s, 0)}^{N_Z}}_{9\cos^2 s + 9\sin^2 s = 9}\, dt\,ds = 72\pi.$$

7.3

STOKES' THM

Let S be a bounded piecewise smooth oriented 2-surface in $\mathbb{R}^3$. Suppose the boundary ∂S of S consists of finitely many piecewise diff'ble (continuously) curves, oriented consistently with S.



Let $F: U^{\text{open}} \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be C^1 vector field, $S \subseteq U \subseteq \mathbb{R}^3$.

THEN:

$$\iint_S \underbrace{(\vec{\nabla \times F})}_{\text{curl } F} \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{s}$$

7.3

p505 Exc # 4

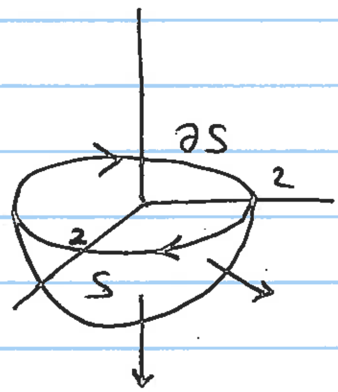
Verify Stokes' Thm

$$F = (2y - z,\ x + y^2 - z,\ 4y - 3x)$$

$S:\quad x^2 + y^2 + z^2 = 4$

$z \le 0$

downward normal.



∂S is a Curve:

clockwise seen from top.

$$\vec{x}(t) = (\overset{x}{2\sin t}, \overset{y}{2\cos t}, \overset{z}{0}) \qquad 0 \le t \le 2\pi$$

$$x'(t) = (2\cos t, -2\sin t, 0)$$

RHS

$$\int \vec{F}\cdot d\vec{s} = \int_0^{2\pi} \underbrace{(4\cos t - 0,\ 2\sin t + 4\cos^2 t - 0,\ 8\cos t - 6\sin t)}_{F(x(t))} \cdot \underbrace{(2\cos t, -2\sin t, 0)\,dt}_{x'(t)dt}$$

$$= \int_0^{2\pi} (8\cos^2 t - 4\sin^2 t - 8\cos^2 t \sin t)\,dt$$

$$= \int_0^{2\pi} 8 \cdot \frac{1 + \cos 2t}{2} - 4 \cdot \frac{1 - \cos 2t}{2} - 8\cos^2 t \sin t\,dt$$

$$= (4 - 2)\cdot 2\pi = 4\pi$$

$$\int_0^{2\pi} \cos 2t\,dt = 0$$

$$\int_0^{2\pi} \cos^2 t \sin t\,dt = 0$$

LHS

S has downward normal (Given)

$$\iint_S \underbrace{\text{curl}\, F}_{\nabla \times F} \cdot d\vec{S} = \iint_S (5, 2, -1) \cdot d\vec{S}$$

$$\text{curl}\, F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2y - z & x + y^2 - z & 4y - 3x \end{vmatrix}$$

$$= (5, 2, -1)$$

$$\underline{X}(s,t) = \left(s, t, -\sqrt{4 - s^2 - t^2}\right) \qquad s^2 + t^2 \leq 4$$

$$\underline{X}_s = \left(1, 0, \frac{+2s}{2\sqrt{4-s^2-t^2}}\right)$$

$$\underline{X}_t = \left(0, 1, \frac{t}{\sqrt{4-s^2-t^2}}\right)$$

$$N = \left(\frac{-s}{\sqrt{4-s^2-t^2}}, \frac{-t}{\sqrt{4-s^2-t^2}}, 1\right)$$

upward normal

Caution

opposite Normals

$$\iint_{\underline{X}} F \cdot d\vec{S} = \iint_{s^2+t^2 \leq 4} (5, 2, -1) \cdot \left(\frac{-s}{\sqrt{}}, \frac{-t}{\sqrt{}}, 1\right)$$

$$= \iint_{s^2+t^2 \leq 4} \frac{-5s - 2t}{\sqrt{4 - s^2 - t^2}} - 1 = -1 \cdot 4\pi = -4\pi$$

PTO

$$-(-4\pi) = -\iint_{\Sigma} \vec{F}\cdot d\vec{S} = \iint_{S} \vec{F}\cdot d\vec{S} = 4\pi$$

since Σ has upward normal, S has downward normal

By Using:

$$\iint_{s^2+t^2\leq 4} \frac{-5s-2t}{\sqrt{4-s^2-t^2}}\, dA_{s,t}$$

use polar $s = r\cos\theta$
$t = r\sin\theta$

$$= \int_0^2 \int_0^{2\pi} \frac{-5r\cos\theta - 2r\sin\theta}{\sqrt{4-r^2}}\, r \, d\theta\, dr = 0$$

since

$$\int_0^{2\pi} \cos\theta\, d\theta = 0 = \int_0^{2\pi} \sin\theta\, d\theta$$