Dec 5

7.1

p468 Exc #7

$$\mathbb{X} = (s\cos t,\ s\sin t,\ s^2) \qquad s \geq 0,\quad 2\pi \geq t \geq 0$$

$$\frac{\partial \mathbb{X}}{\partial s} = (\cos t,\ \sin t,\ 2s)$$

$$\frac{\partial \mathbb{X}}{\partial t} = (-s\sin t,\ s\cos t,\ 0)$$

$$N = (-2s^2\cos t,\ -2s^2\sin t,\ \underbrace{s\cos^2 t + s\sin^2 t}_{s})$$

$$= s\cdot(-2s\cos t,\ -2s\sin t,\ 1)$$

a) If $s=0$, then $N=0$

If $s \neq 0$, then surface is smooth (since then $N \neq 0$).

Eq for tangent plane $(1, \sqrt{3}, 4)$

$$\mathbb{X}(2, \tfrac{\pi}{3}) = (1, \sqrt{3}, 4)$$

$$\begin{array}{cc} \shortparallel & \shortparallel \\ s & t \end{array}$$

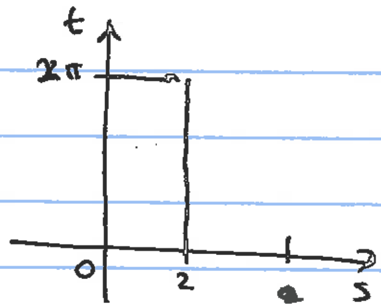
$$N(2, \tfrac{\pi}{3}) = \left(-8\cdot\tfrac{1}{2},\ -8\cdot\tfrac{\sqrt{3}}{2},\ 2\right)$$

$$= (-4,\ -4\sqrt{3},\ 2)$$

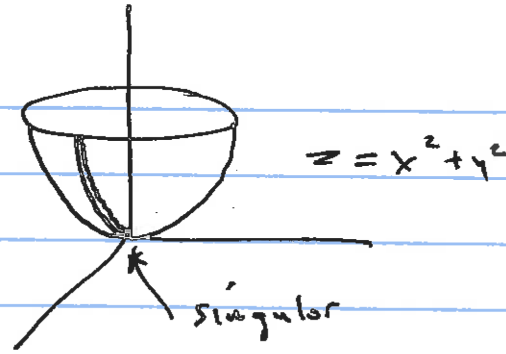
Tangent plane:

$$(-4, -4\sqrt{3}, 2)\cdot\left[(x,y,z) - (1,\sqrt{3},4)\right] = 0$$

$$-4x - 4\sqrt{3}y + 2z = -4 - 12 + 8 = -8$$



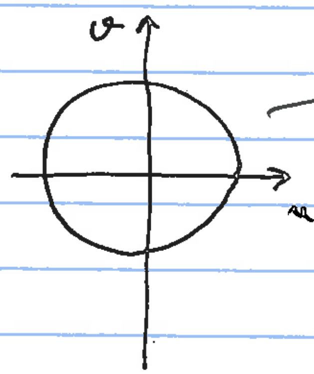
(b)

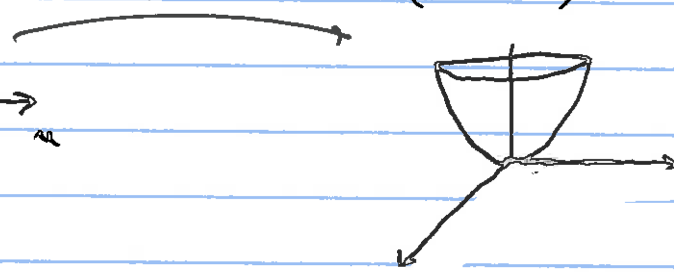


(c) $z = f(x,y) = x^2+y^2$

Another parametrization (not $\left\{ \begin{matrix} \text{smooth} \\ \text{singular} \end{matrix} \right\}$ at $(0,0,0)$)

$$Y(u,v) = (u,\ v,\ u^2+v^2)$$





$X(s,t)$ is singular at $(0,0,0)$ $\quad N_X(0,0) = \vec{0}$

$Y(u,v)$ is regular at $(0,0,0)$ $\quad N_Y \neq \vec{0}$

$$\frac{\partial Y}{\partial u} \times \frac{\partial Y}{\partial v} = (1,0,2u) \times (0,1,2v) = (-2u,-2v,1)$$

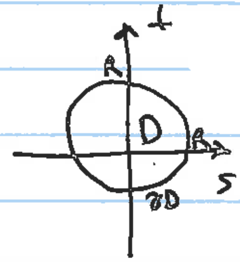
(*) Important:

- $X = (s, t, f(s,t))$ parametrizes the graph surface $z = f(x,y)$
- $Y = (s, g(s,t), t)$ " $y = g(x,z)$
- $Z = (h(s,t), s, t)$ " $x = h(y,z)$

Spheres :

(1) $\Sigma(s, t, \sqrt{R^2 - s^2 - t^2})$

$s^2 + t^2 \leq R$



$$\frac{\partial \Sigma}{\partial s} = \left(1, 0, \frac{-2s}{2\sqrt{R^2 - s^2 - t^2}}\right)$$

$$\frac{\partial \Sigma}{\partial t} = \left(0, 1, \frac{-2t}{2\sqrt{R^2 - s^2 - t^2}}\right)$$

$$\frac{\partial \Sigma}{\partial s} \times \frac{\partial \Sigma}{\partial t} \quad N = \left(\frac{s}{\sqrt{R^2 - s^2 - t^2}}, \frac{t}{\sqrt{R^2 - s^2 - t^2}}, 1\right) \neq \vec{0},$$

if $R^2 - s^2 - t^2 > 0$.

BUT: N is undefined if $\underbrace{R^2 = s^2 + t^2}_{\text{Boundary of } D} \longrightarrow$ equator

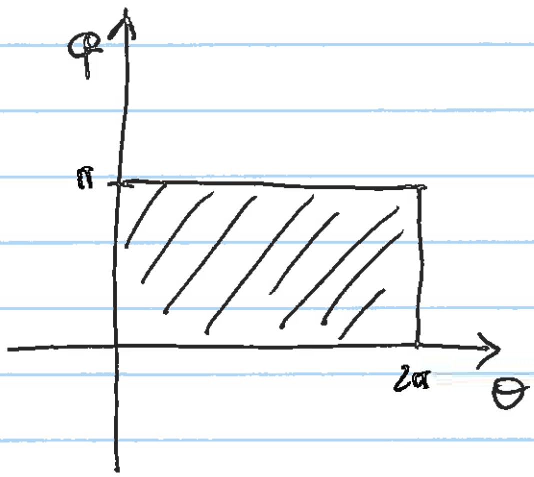
(2) $\Sigma(r, \theta) = \left(r\cos\theta, r\sin\theta, \sqrt{R^2 - r^2}\right)$ singular along equator,

$0 \leq \theta \leq 2\pi \qquad 0 \leq r \leq R$ and when $r = 0$. (north pole)

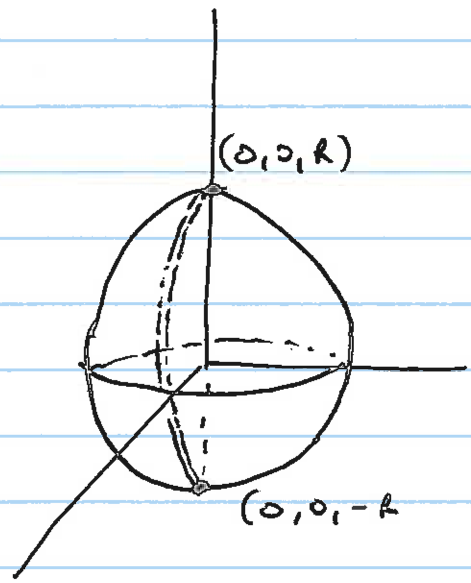
(3) $\Sigma(\theta, \varphi) = (R \sin\varphi \cos\theta, R \sin\varphi \sin\theta, R\cos\varphi)$

R fixed

$0 \leq \theta \leq 2\pi \qquad 0 \leq \varphi \leq \pi$

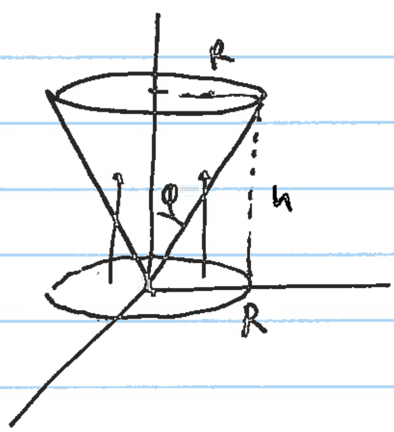


$Z(\theta, \varphi)$ is singular at $(0,0,\pm R)$



Ex

Cone



$$z = \frac{h}{R}\sqrt{x^2+y^2}$$

Domain

- $X(s,t) = \left(s,\ t,\ \frac{h}{R}\sqrt{s^2+t^2}\right)$ $\qquad 0 \le s^2+t^2 \le R^2$

- $Y(r,\theta) = \left(r\cos\theta,\ r\sin\theta,\ \frac{h}{R}r\right)$

spherical $\varphi = \varphi_0$ constant

- $Z(\rho,\theta) = \left(\rho\frac{R}{\sqrt{R^2+h^2}}\cos\theta,\ \rho\frac{R}{\sqrt{R^2+h^2}}\sin\theta,\ \rho\frac{h}{\sqrt{R^2+h^2}}\right)$

$\tan\varphi_0 = \frac{R}{h}$ fixed $\Longrightarrow$ $\sin\varphi_0 = \frac{R}{\sqrt{R^2+h^2}}$ $\quad \cos\varphi_0 = \frac{h}{\sqrt{R^2+h^2}}$

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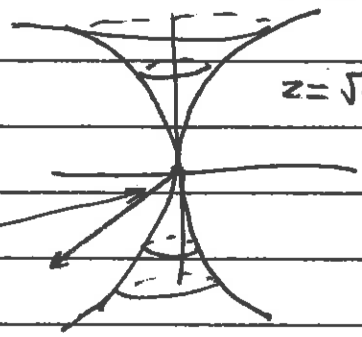
p 468 4c $\quad \mathbf{X}(s,t) = (s^2 \cos t,\ s^2 \sin t,\ s)$ $\qquad -3 \le s \le 3$, $\ 0 \le t \le 2\pi$



$z = \sqrt{r}$

$x^2 + y^2 = z^4.$

$r^2 = z^4$

$s = 0$
pinched
not regular
not smooth.

circle of radius 4 at
height ± 2

circle of radius 9 at
height ± 3

(a) $\dfrac{\partial \mathbf{X}}{\partial s} = (2s\cos t,\ 2s \sin t,\ 1)$

$\dfrac{\partial \mathbf{X}}{\partial t} = (-s^2 \sin t,\ s^2 \cos t,\ 0)$

$\dfrac{\partial \mathbf{X}}{\partial s} \times \dfrac{\partial \mathbf{X}}{\partial t} = N = (-s^2 \cos t,\ -s^2 \sin t,\ 2s^3) \neq 0$ if $s \neq 0$.

(b) Tangent plane at $(1,0,-1) = \mathbf{X}(\underset{s}{-1}, \underset{t}{0})$

$N(\underset{s}{-1}, \underset{t}{0}) = (-1, 0, -2)$

$(-1,0,-2)\cdot[(x,y,z) - (1,0,-1)] = 0$

$-x - 2z = 1$