Dec 2

(1)

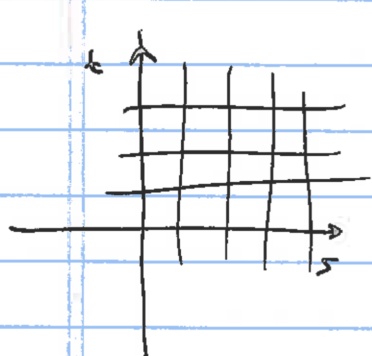
7.1

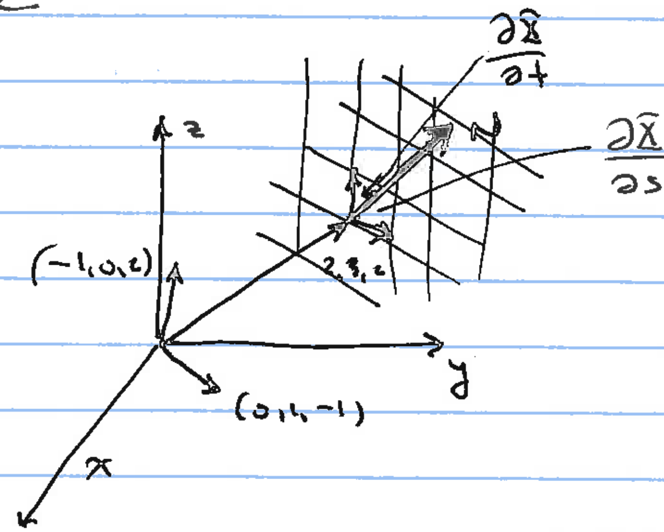
Ex

$\bar{X}(s,t) = (2,3,2) + s(0,1,-1) + t(-1,0,2) = (x,y,z)$

Recall Chap I

parametrized plane





$\bar{X} : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$

s, t $\qquad$ x, y, z

$\frac{\partial \bar{X}}{\partial s} = (0,1,-1)$

$\frac{\partial \bar{X}}{\partial t} = (-1,0,2)$

$$N = \frac{\partial \bar{X}}{\partial s} \times \frac{\partial \bar{X}}{\partial t} = \begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= (2,1,1)$$

$N \perp$ plane.

$\|N\| = \sqrt{4+1+1} = \sqrt{6}$ = area of parallelogram whose sides are $(0,1,-1), (-1,0,2)$

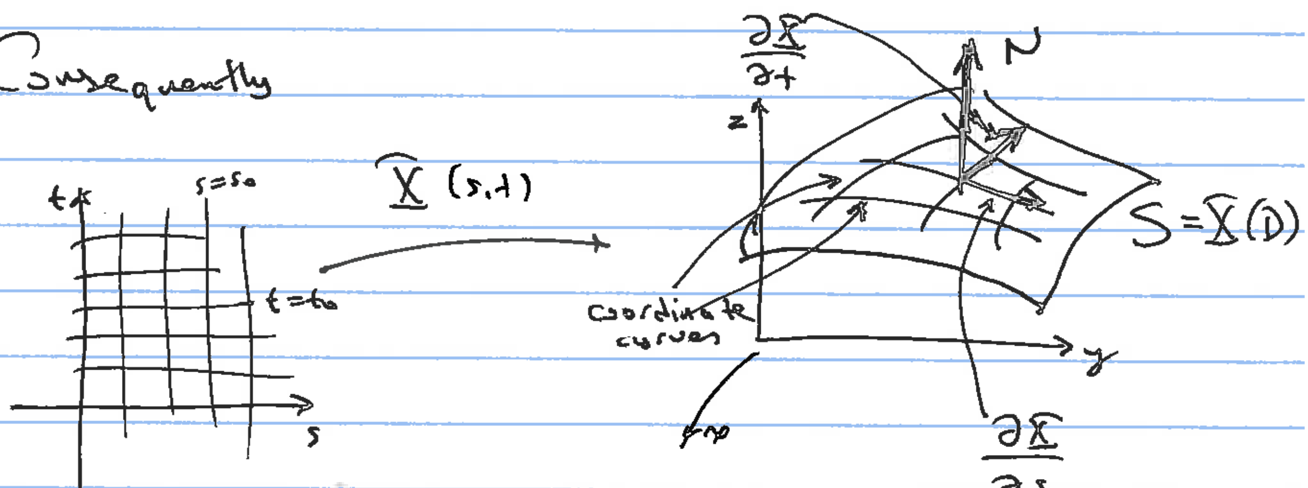
Defn Let $\Phi : D^{open} \subseteq \mathbb{R}^2_{s,t} \longrightarrow \mathbb{R}^3_{x,y,z}$ be 1-1 except possibly at boundary of D.

1. $\Phi(D) = S$ is called a parametrized surface

2. Φ is called a smooth/diff'ble surface if Φ diff'ble and
$$\frac{\partial \Phi}{\partial s} \times \frac{\partial \Phi}{\partial t} = N(s,t) \neq 0 \quad \text{on } D$$

3. $\Phi(s_0, t)$, $\Phi(s, t_0)$ are called coordinate curves

Consequently



$$N = \frac{\partial \Phi}{\partial s} \times \frac{\partial \Phi}{\partial t} \perp S \text{ at } \Phi(s_0, t_0)$$

$N(s_0, t_0) \perp$ Tangent plane at $\Phi(s_0, t_0)$

$$N(s_0, t_0) \cdot \Big[(x,y,z) - \underbrace{\Phi(s_0, t_0)}_{p_0} \Big] = 0$$

$$\|N\| = \left\| \frac{\partial \mathbf{X}}{\partial s} \times \frac{\partial \mathbf{X}}{\partial t} \right\|$$

$$dS = \|N\| \, ds\,dt$$

$$\text{Area} = \iint_{\mathbf{X}(D)=S} 1 \, dS = \iint_{D} \|N\| \, ds\,dt$$

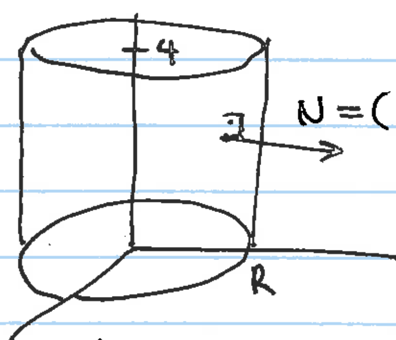
<u>Ex 1</u> $\mathbf{X}(s,t) = (R\cos s, R\sin s, t)$

$0 \le s \le 2\pi$

$0 \le t \le 4$.

cylinder <u>only side</u>

open top & bottom



$$\frac{\partial \mathbf{X}}{\partial s} = (-R\sin s, R\cos s, 0)$$

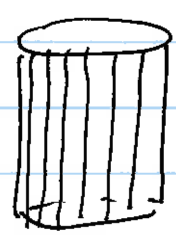
$$\frac{\partial \mathbf{X}}{\partial t} = (0, 0, 1)$$

$$\frac{\partial \mathbf{X}}{\partial s} \times \frac{\partial \mathbf{X}}{\partial t} = N = (R\cos s, R\sin s, 0)$$

Coordinate curves $\mathbf{X}(s, t_0) \rightarrow$

height t_0

$\mathbf{X}(s_0, t)$



$$\|N\| = \sqrt{R^2\cos^2 s + R^2\sin^2 s} = \sqrt{R^2} = R.$$

$$\text{Area} = \iint_D \|N\|\,ds\,dt = \int_0^{2\pi}\int_0^4 R\,ds\,dt$$

$$= 8\pi R.$$

Tangent plane at $\bar{X}(\overset{s}{\overset{\|}{0}},\overset{t}{\overset{\|}{2}}) = (R,0,2)$?

$$N(0,2) = (R,0,0)$$

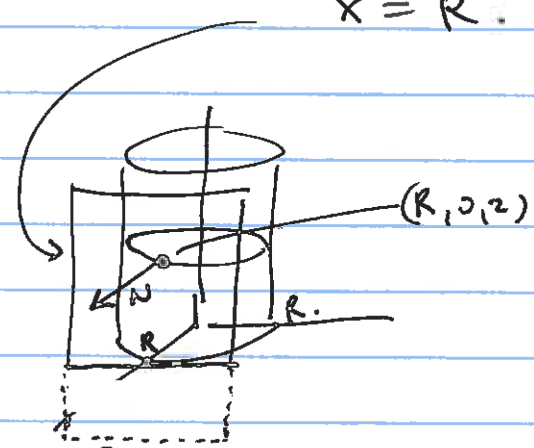
eqn Tangent plane :

$$(R,0,0)\cdot\left[(x,y,z)-(R,0,2)\right] = 0.$$

$$Rx = R^2.$$

$$x = R.$$

Tangent plane.



$N = (R,0,0)$