6.1

Reparametrizations

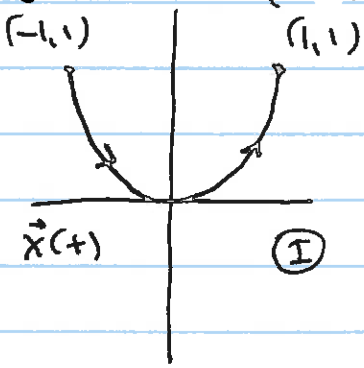
$$\left(\text{First coord}\right)^2 = \left(\text{2nd coord.}\right)$$

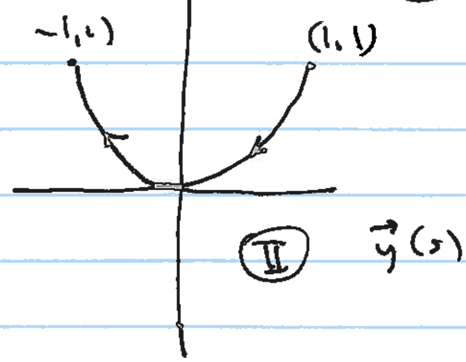
$\vec{x}(t) = (t, t^2)$ $\quad -1 \le t \le 1$ $\quad$ I

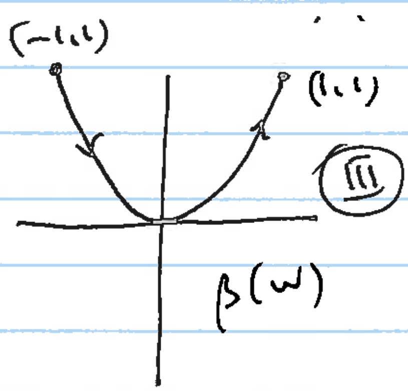
$\vec{y}(s) = (1-s, (s-1)^2)$ $\quad 0 \le s \le 2$ $\quad$ II $\quad t = 1-s$

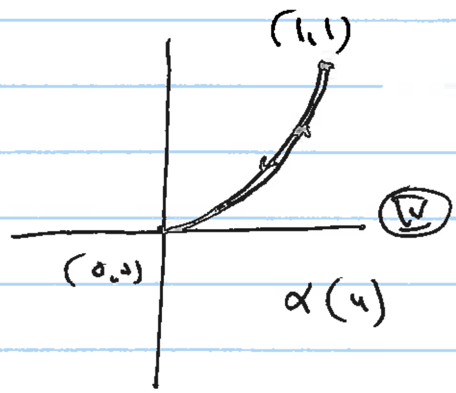
$\alpha(u) = (u^2, u^4)$ $\quad -1 \le u \le 1$ $\quad$ IV $\quad t = u^2$

$\beta(w) = (w^3, w^6)$ $\quad -1 \le w \le 1$ $\quad$ III $\quad t = w^3$









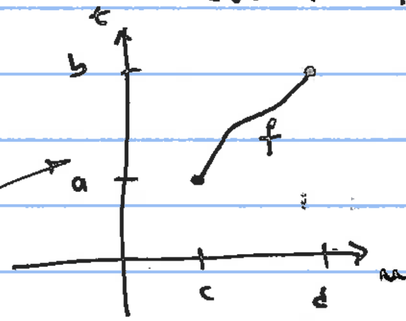
Scalar $\displaystyle\int_{\text{curve}} f \cdot ds$ $\quad$ f real valued $\quad$ I, II, III $\longrightarrow$ same answer

Vector $\displaystyle\int_{\text{curve}} \vec{F} \cdot d\vec{s}$ $\quad$ F vector valued. $\quad$ I, III $\longrightarrow$ same

II $\longrightarrow$ opposite sign.

IV is not good since it is not tracing the same path.

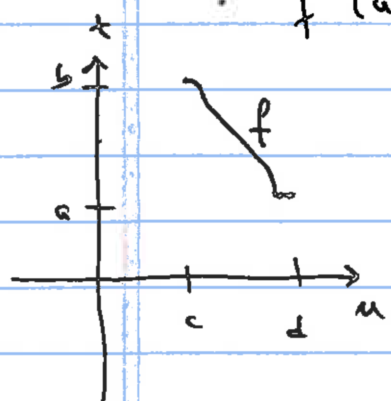
<u>Defn</u> Let $\vec{x}(t) : [a,b] \longrightarrow \mathbb{R}^n$ be a parametrized curve. Let



$t = f(u)$ be a substitution s.t.
$$\begin{cases} f : [c,d] \to [a,b] \\ 1-1,\ \text{onto} \\ \text{diff'ble} \\ f'(u) \neq 0 \end{cases}$$

Then $\vec{x}(t) = \vec{x}(f(u)) = \vec{y}(u)$ is called a re-parametrization of $x(t)$.

- $f'(u) > 0$ then $\vec{y}(u)$ & $\vec{x}(t)$ trace the path in the same direction
- $f'(u) < 0$ then $\vec{y}(u)$ & $\vec{x}(t)$ trace the path in opposite directions.



orientation

$$\vec{x}(f(u)) = y(u) : [c,d] \longrightarrow \mathbb{R}^n.$$

<u>Prop</u> Let $x(t)$ and $y(u) = x(f(u))$ be reparametrizations of each other.

(i) For any real valued function g (defined along the curve)

Scalar
$$\int_{\vec{x}} g \cdot ds = \int_a^b g(x(t))\,\|x'(t)\|\,dt = \int_{\vec{y}} g \cdot ds =$$
$$= \int_c^d g(y(u))\,\|y'(u)\|\,du$$

(ii) <u>next page</u>

(ii) For any vector valued function G

Vector line integral

$$\int_{\vec{x}} G \cdot d\vec{s} = \int_a^b G(x(t)) \cdot x'(t)\,dt =$$

$$\pm \int_{\vec{y}} G \cdot ds = \int_c^d G(y(u)) \cdot y'(u)\,du$$

$+$ if $x(t)$, & $y(u)$ traverse the path in the same direction.

$-$ if $x(t)$, & $y(u)$ traverse the path in opposite directions.

6.1 Exc # 24 p 427.

$$\int_C (x^2-y)\,dx + (x-y^2)\,dy$$

C line segment from $(1,1)$, to $(3,5)$

Parametrize

$$\vec{x}(t) = (1-t)(1,1) + t(3,5) \qquad 0 \le t \le 1$$

$$= (1-t+3t,\ 1-t+5t)$$

$$= (\underbrace{1+2t}_{x},\ \underbrace{1+4t}_{y})$$

$$x = 1+2t \qquad dx = 2\,dt$$

$$y = 1+4t \qquad dy = 4\,dt$$

$$= \int_0^1 \left(\left((1+2t)^2 - (1+4t)\right) 2\,dt + \left((1+2t) - (1+4t)^2\right) 4\,dt\right)$$

$$= \int_0^1 (\cancel{1} + \cancel{4t} + 4t^2 - \cancel{1} - \cancel{4t})\, 2\,dt$$

$$+ (\cancel{1} + 2t - \cancel{1} - 8t - 16t^2)\, 4\,dt$$

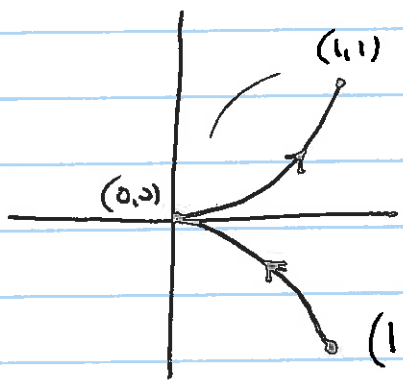
$$= \int_0^1 (8t^2 - 24t - 64t^2)\,dt$$

$$= \frac{8}{3}t^3 - 12t^2 - \frac{64}{3}t^3 \Big|_0^1$$

$$= \frac{8}{3} - 12 - \frac{64}{3} = -12 - \frac{56}{3} = \frac{-92}{3}.$$

p427 # 26 $\displaystyle\int_C x^2 y\,dx - xy\,dy$

$y^2 = x^3$ from $(1,-1)$ to $(1,+1)$



$y = \sqrt{x^3} = x^{\frac{3}{2}}$

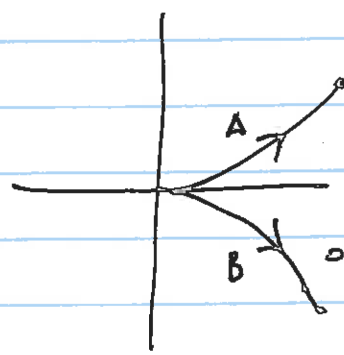
How do we parametrize?

I option

$0 \le t \le 1$ $\quad (\underbrace{t}_{x}, \underbrace{t^{3/2}}_{y})$ A

$y^2 = x^3$

$0 \le t \le 1$ $\quad (t, -t^{3/2})$ B.



Caution opposite direction of the question.

$$\int x^2 y\,dx - xy\,dy = + \int_A \quad - \int_B$$

II

$(\underbrace{u^2}_{x}, \underbrace{u^3}_{y}) \qquad -1 \le u \le 1$

$y^2 = x^3$

$x = u^2 \qquad dx = 2u\,du$

$y = u^3 \qquad dy = 3u^2\,du$

$$\int_C x^2 y\,dx - xy\,dy = \int_{-1}^{1} u^7 \cdot 2u\,du - u^5 \cdot 3u^2\,du = \int_{-1}^{1} (2u^8 - 3u^7)\,du = \frac{4}{9}.$$

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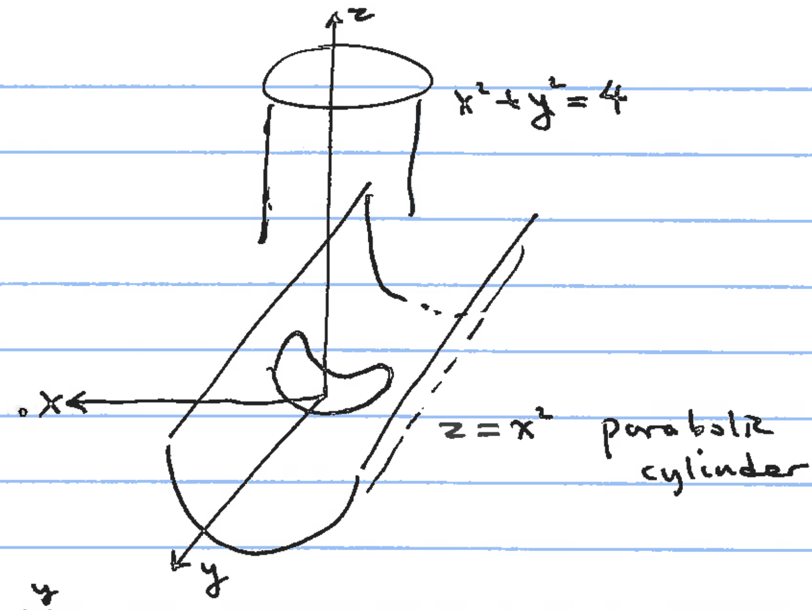
#32

curve of intersection $\begin{cases} z = x^2 \\ x^2+y^2 = 4 \end{cases}$



$$\Big(\underbrace{\overbrace{2\cos t}^{x},\ \overbrace{2\sin t}^{y}}_{\substack{x^2+y^2=4 \\ \text{top view}}},\ \underbrace{4\cos^2 t}_{z=x^2}\Big) \qquad 0 \le t \le 2\pi$$

$$\int_C z\,dx + x\,dy + y\,dz$$

$$= \int_0^{2\pi} \underbrace{(4\cos^2 t)}_{z}\underbrace{(-2\sin t\,dt)}_{dx} + \underbrace{(2\cos t)}_{x}\underbrace{(2\cos t)}_{dy}dt + \overbrace{2\sin t}^{y}\cdot\ d$$

$$\underbrace{(-8\cos t\sin t}_{dz}$$

$$= \int_0^{2\pi} 4\cos^2 t\,dt = \int_0^{2\pi} 2(1+\cos 2t)\,dt = 4\pi$$

$$\int_0^{2\pi} \cos^2 t\sin t\,dt = 0 = \int_0^{2\pi} \cos t\sin^2 t\,dt$$