Nov 16 ①

3.4 **Defn** Let $F : U \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ be diff'ble.
We define Curl $F : U \subseteq \mathbb{R}^3 \to \mathbb{R}^3$ by

$$\operatorname{curl} F = \nabla \times F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

(where $F = (F_1, F_2, F_3)$)

Ex $F(x,y,z) = (x+3y, xyz, yz)$

$$\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ x+3y & xyz & yz \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y} yz - \frac{\partial}{\partial z} xyz, \frac{\partial}{\partial z}(x+3y) - \frac{\partial}{\partial x} yz, \frac{\partial}{\partial x} xyz - \frac{\partial}{\partial y} x+3y \right)$$

$$= (z - xy, 0, yz - 3)$$

#10 p 235 Ex $F = (\cos yz - x) i + (\cos xz - y) j + (\cos xy - z) k$

$$\operatorname{curl} F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \cos yz - x & \cos xz - y & \cos xy - z \end{vmatrix}$$

$$= (-x \sin xy + x \sin xz, -y \sin yz + y \sin xy, -z \sin xz + z \sin yz)$$

<u>Prop</u> if $f : U^{\text{open}} \subseteq \mathbb{R}^3 \to \mathbb{R}^1$ is <u>twice continuously diff'ble</u>, then

$$\nabla \times \nabla f = 0$$

$$\text{curl}(\text{grad } f) = 0.$$

<u>Proof</u> direct calculation

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\nabla \times \nabla f = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$$

$$= \Big(\underbrace{\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}}_{0}, \underbrace{\frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z}}_{0}, \underbrace{\frac{\partial^2 f}{\partial y \partial x} - \frac{\partial^2 f}{\partial x \partial y}}_{0} \Big)$$

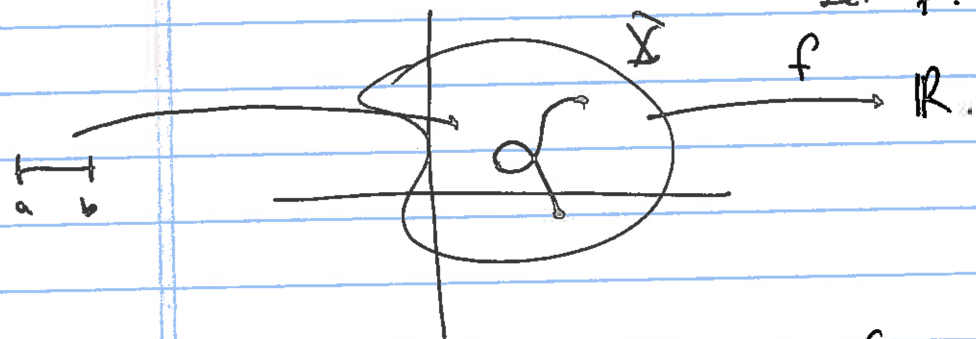
since f is twice continuously diff'ble.

* See page 137 Thm 4.3 of the textbook.

Chap VI

6.1 Defn Let $\vec{x} : [a,b] \longrightarrow X \subseteq \mathbb{R}^n$ be a diff'ble curve.

Let $f : X \longrightarrow \mathbb{R}$ continuous



Then one defines $\displaystyle\int_{\vec{x}} f\,ds = \int_a^b f(\vec{x}(t))\underbrace{\|x'(t)\|\,dt}_{ds}$

(scalar line integral)

p 426 Exc #2

$f(x,y,z) = xyz$

$\vec{x}(t) = (t, 2t, 3t)$ $\quad 0 \le t \le 2$

line

$$\int_a^b f(\vec{x}(t))\,\|x'(t)\|\,dt = \int_0^2 f(t, 2t, 3t)\sqrt{14}\,dt$$

$\vec{x}' = (1, 2, 3)$

$\|x'\| = \sqrt{1+4+9} = \sqrt{14}$

$$\int_0^2 6t^3\sqrt{14}\,dt = \frac{6t^4}{4}\sqrt{14}\Big|_0^2 = 24\sqrt{14}.$$

p 426 #4. $f(x,y,z) = 3x + xy + z^3$

$\vec{x}(t) = (\underbrace{\cos 4t}_{x}, \underbrace{\sin 4t}_{y}, \underbrace{3t}_{z})$ helix

$0 \le t \le 2\pi$

$$\int_x f\,ds = \int_a^b f(x(t))\,\|x'(t)\|\,dt$$

$x'(t) = (-4\sin 4t, 4\cos 4t, 3)$

$\|x'(t)\| = 5$

$f(x(t)) = 3\cos 4t + \cos 4t \sin 4t + 27t^3$

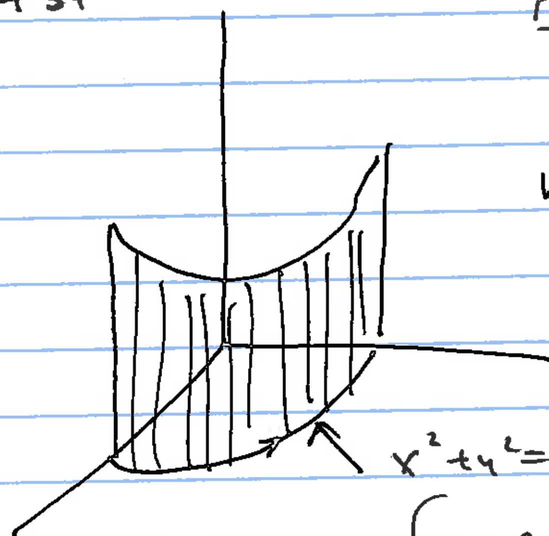
$$= \int_0^{2\pi} \Big(3\cos 4t + \underbrace{\cos 4t \sin 4t}_{\frac{1}{2}\sin 8t} + 27t^3\Big)\, 5\, dt$$

$$= \left(\frac{3\sin 4t}{4} + \frac{1}{16} \cdot -\cos 8t + \frac{27}{4} t^4 \Bigg|_0^{2\pi} \right) \cdot 5$$ ✱ Corrected

$$= 5\left(0 + 0 + \frac{27}{4}(2\pi)^4\right) = 104\pi^4 \cdot 5 = 520\pi$$

p427 #34

Area of the fence



height

$h(x,y) = 10 - x - y.$

$x^2+y^2=25, \quad x \geq 0, \quad y \geq 0$

$\vec{x}(t) = (\overbrace{5\cos t}^{x}, \overbrace{5\sin t}^{y}) \qquad 0 \leq t \leq \frac{\pi}{2}$

$$\int_{\vec{x}} h\,ds = \int_0^{\pi/2} h(5\cos t, 5\sin t) \cdot 5 \cdot dt$$

$x'(t) = (-5\sin t, 5\cos t)$

$|x'(t)| = 5$

$$\underline{\underline{\text{Area}}} = \int_0^{\pi/2} \Big(10 - (5\cos t) - (5\sin t)\Big)\, 5 \cdot dt$$

$$= 5 \cdot \left(10t - 5\sin t + 5\cos t\right)\Big|_0^{\pi/2}$$

$$= 5 \cdot (5\pi - 5 + 0) - (0 - 0 + 5)$$

$$= 5(5\pi - 10)$$

$$= 25\pi - 50.$$

	MT I	MT II	Average
Student X	92 A	92 A-	92 A-
A/A- cut	90	95	92.5

4 quizzes	$\frac{4}{5} \times 15\%$	12 %	as of Nov 14
2 MT.		50 %	
(10 Hw / 12) of 10%		8.3	
		70.3 %	