Nov 10

0

Review Session This evening 11/10/16
205 MLH 6:30 – 7:30

Midterm 2 11/11/16 10:30 – 11:20

- 2.6
- 3.1, 3.2
- 4.1, 4.2, 4.3
- 5.1, 5.2, 5.3, 5.4

+ Everything else before

Final Exam Friday Dec 16, 2016
3pm – 5 pm.

Plan:

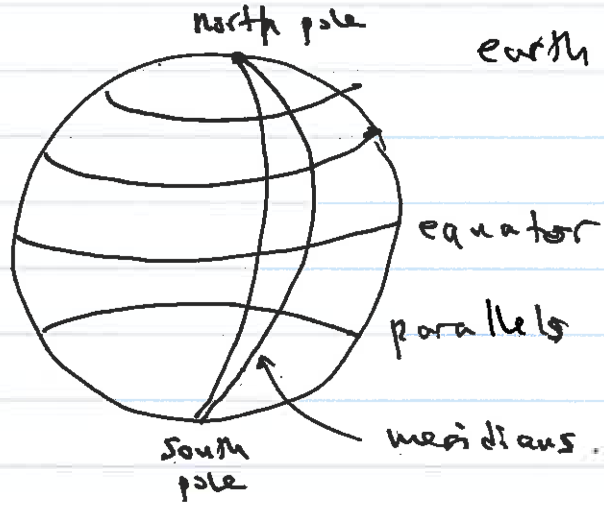
Today 5.5; to finish Chap V.

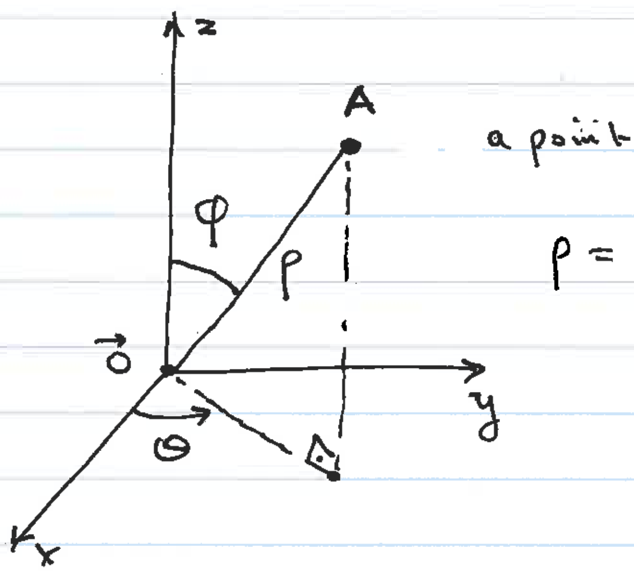
next 3 weeks

3.1/3.2	~1 hr
6.1	~2 hr
6.2	2
6.3	2
7.1	2
7.2	2
7.3	2
	13 hrs.

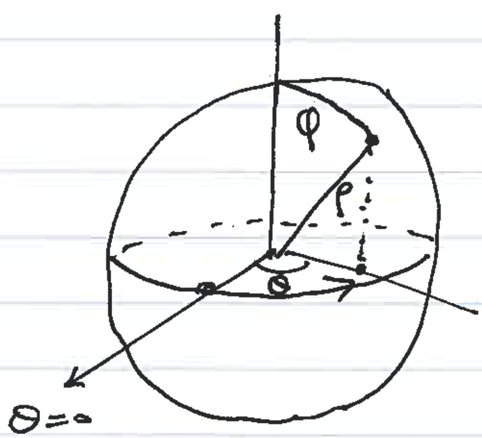
$\underbrace{\text{12 days} - \tfrac{1}{2}\text{ quiz 5}}_{\text{remaining days}}$

Spherical Coordinates





ρ = distance from O to A



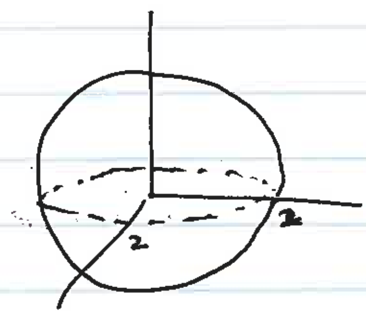
$$\begin{cases} x = \rho \sin\varphi \cos\theta \\ y = \rho \sin\varphi \sin\theta \\ z = \rho \cos\varphi \end{cases}$$

$$\begin{cases} \rho = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\dfrac{y}{x} + k\pi \\ \varphi = \cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}} \end{cases}$$

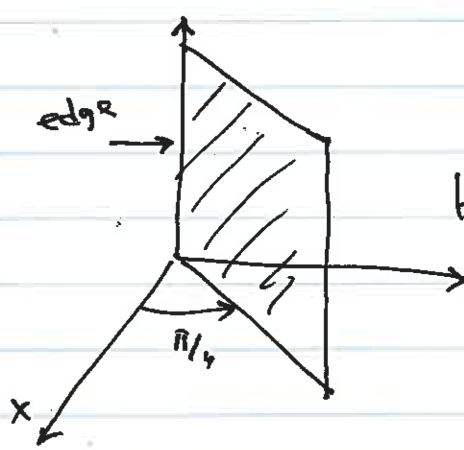
$$\frac{\partial(x,y,z)}{\partial(\rho,\varphi,\theta)} = \cdots = \rho^2 \sin\varphi .$$

Examples of
Fundamental surfaces in spherical coordinates:

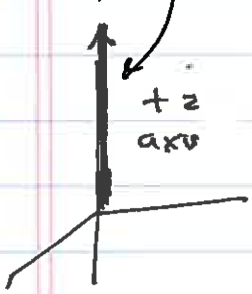
$\rho = 2$ Sphere of radius 2



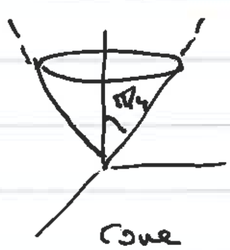
$\theta = \frac{\pi}{4}$ Half plane.



$\varphi = 0$

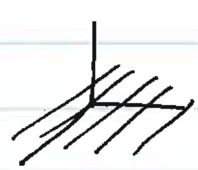


$\varphi = \frac{\pi}{4}$



$\varphi = \frac{\pi}{2}$

xy-plane



$\varphi = \pi$

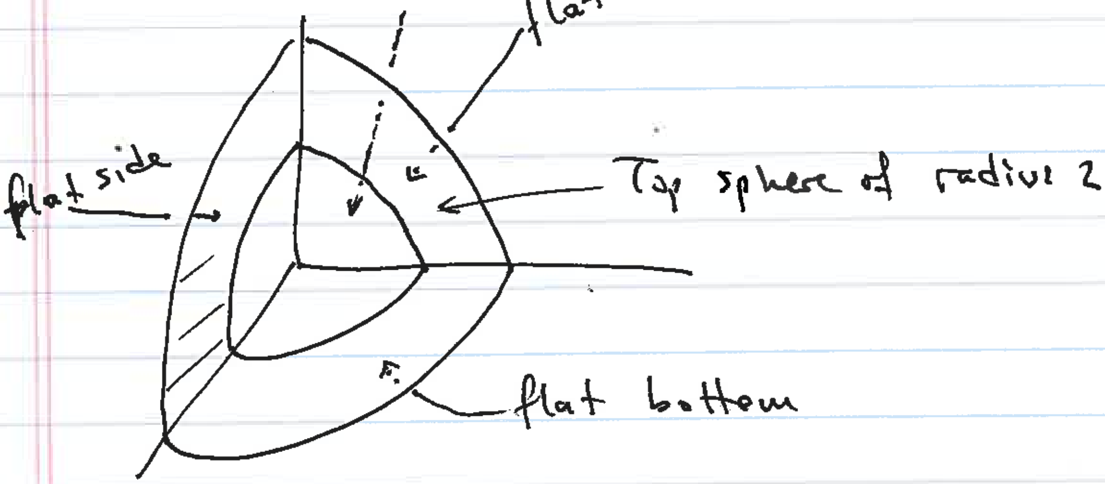
Ex 1

Let W be the region between the spheres

$$x^2+y^2+z^2=1$$
$$x^2+y^2+z^2=4$$

in the first octant $x \geq 0,\ y \geq 0,\ z \geq 0$

Calculate
- Volume
- $\iiint_W x \, dV$



$$1 \leq \rho \leq 2$$
$$0 \leq \varphi \leq \frac{\pi}{2}$$
$$0 \leq \theta \leq \frac{\pi}{2}$$

Jacobian

$$\text{Volume} = \int_0^{\pi/2} \int_0^{\pi/2} \int_1^2 1 \cdot \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

Since all bounds are constant & integrand is factorable into factors of one variable each. ④

$$= \underbrace{\left(\int_0^{\pi/2} 1 \cdot d\theta\right)}\left(\int_0^{\pi/2} \sin\varphi \, d\varphi\right)\underbrace{\left(\int_1^2 \rho^2 \, d\rho\right)}$$

$$= \left(\frac{\pi}{2}\right)\left(-\cos\varphi \Big|_0^{\pi/2}\right)\left(\frac{1}{3}\rho^3 \Big|_1^2\right)$$

$$= \frac{\pi}{2}\left(-\cancel{\cos\frac{\pi}{2}}^{\,0} + \underbrace{\cos 0}_{1}\right)\left(\frac{1}{3}(2^3 - 1^3)\right)$$

Volume $= \dfrac{7\pi}{6}$.

(b) $$\iiint_W x \, dV = \int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \underbrace{\rho\sin\varphi\cos\theta}_{x} \; \underbrace{\rho^2 \sin\varphi}_{\det DT} \, d\rho \, d\varphi \, d\theta$$

$$= \left(\int_0^{\pi/2} \cos\theta \, d\theta\right)\left(\int_0^{\pi/2} \sin^2\varphi \, d\varphi\right)\left(\int_1^2 \rho^3 \, d\rho\right)$$

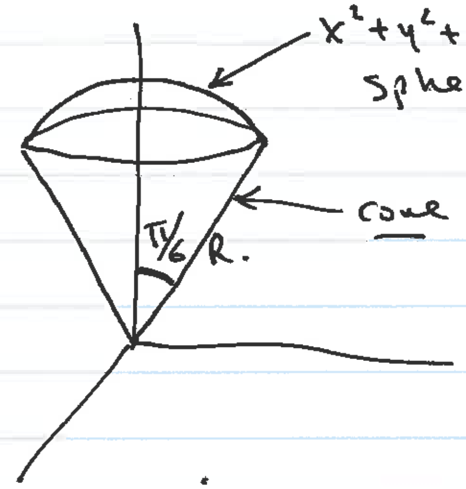
$$= \left(\sin\theta \Big|_0^{\pi/2}\right)\left(\int_0^{\pi/2} \frac{1-\cos 2\varphi}{2} \, d\varphi\right)\left(\frac{\rho^4}{4}\Big|_1^2\right)$$

$$= 1 \cdot \left(\frac{\varphi}{2} - \frac{\sin 2\varphi}{4}\right)\Bigg|_{\varphi=0}^{\varphi=\pi/2} \quad \frac{15}{4}$$

$$= \frac{15}{4}\left[\frac{\pi}{4}\right] = \frac{15\pi}{16}.$$

Volume of an ice cream cone

Ex 2



cone $z = c \cdot \sqrt{x^2+y^2}$

$(c = \cot \frac{\pi}{6})$

$$0 \le \rho \le R.$$
$$0 \le \varphi \le \pi/6$$
$$0 \le \theta \le 2\pi$$

$$V = \int_0^{2\pi} \int_0^{\pi/6} \int_0^R 1 \cdot \rho^2 \sin\varphi \cdot d\rho \, d\varphi \, d\theta$$

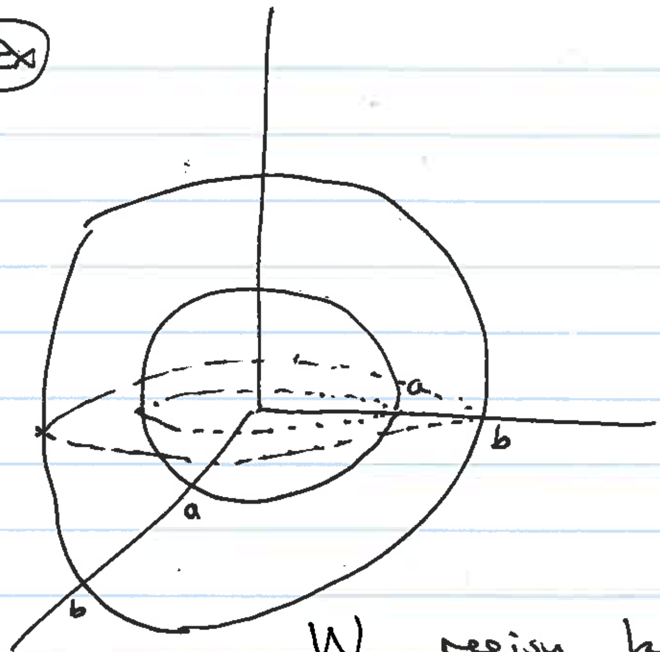
$$= \left(\int_0^{2\pi} d\theta \right) \left(\int_0^{\pi/6} \sin\varphi \right) \left(\int_0^R \rho^2 \, d\rho \right)$$

$$= 2\pi \cdot \left(-\cos\varphi \Big|_0^{\pi/6} \right) \cdot \frac{R^3}{3}$$

$$= \frac{2\pi}{3} R^3 \left(1 - \cos\frac{\pi}{6} \right)$$

$$= \frac{2\pi}{3} R^3 \left(1 - \frac{\sqrt{3}}{2} \right)$$

Ex



$a \le \rho \le b$

$0 \le \theta \le 2\pi$

$0 \le \varphi \le \pi$

W region between $\left\{\right.$ spheres of radii a & b.

$$\iiint_W (x^2+y^2+z^2)\, dV$$

$$= \int_0^{2\pi} \int_0^{\pi} \int_a^b \rho^2 \quad \rho^2 \sin\varphi \cdot d\rho\, d\varphi\, d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right)\left(\int_0^{\pi} \sin\varphi\, d\varphi\right)\left(\int_a^b \rho^4\, d\rho\right)$$

$$= 2\pi \cdot \left(-\cos\varphi \Big|_0^{\pi}\right) \cdot \frac{b^5-a^5}{5}$$

$$\underbrace{-\underbrace{\cos\pi}_{-1} + \cos 0}_{2}$$

$$= \frac{4\pi}{5}(b^5-a^5)$$