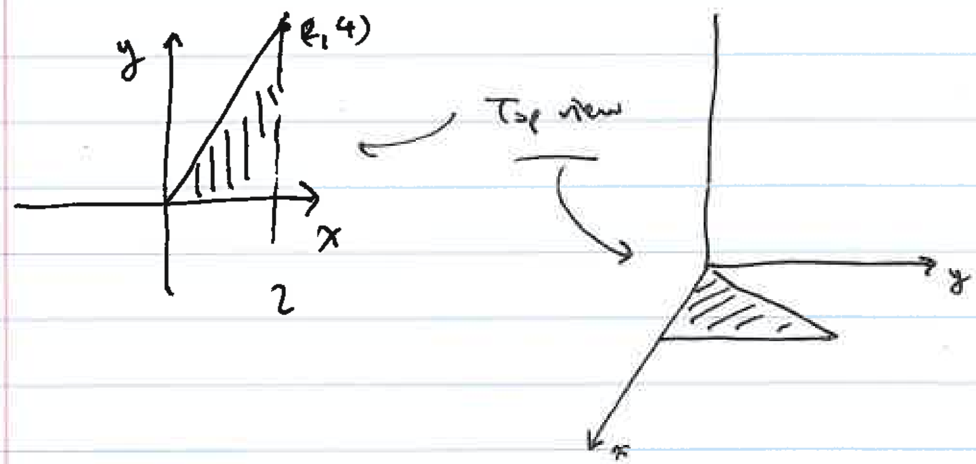
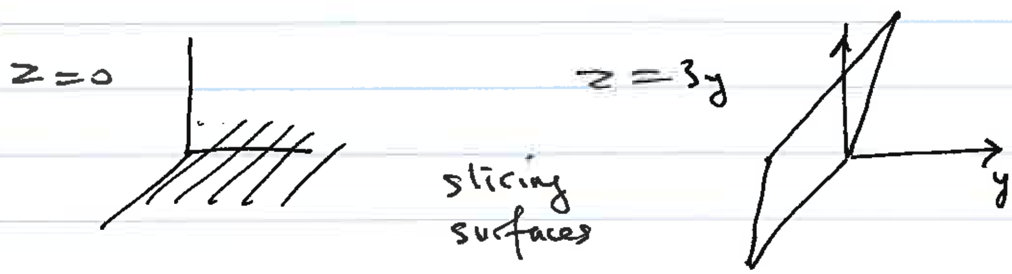
Caution ordered according to problem #5.

Practice 2b) $\int_0^2 \int_0^{2x} \int_0^{3y} \cdots \, dz\, dy\, dx$

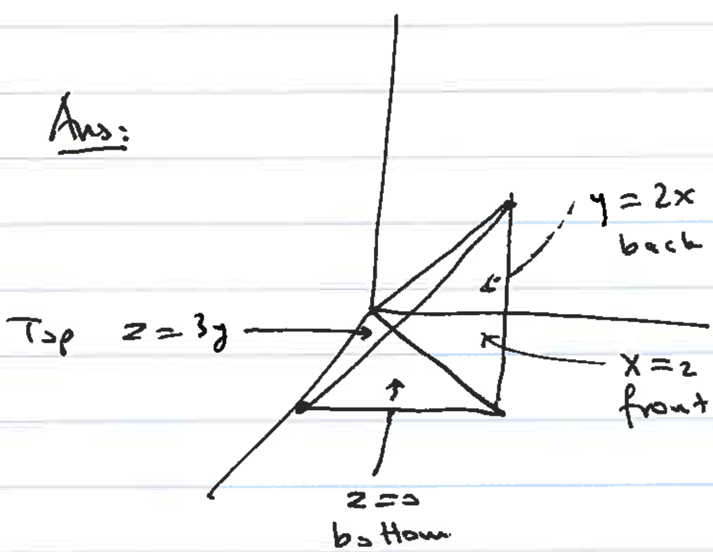
$$\left.\begin{array}{l} 0 \le x \le 2 \\ 0 \le y \le 2x \end{array}\right\} \text{ If one looks from } z\text{-axis direction}$$

$$0 \le z \le 3y$$



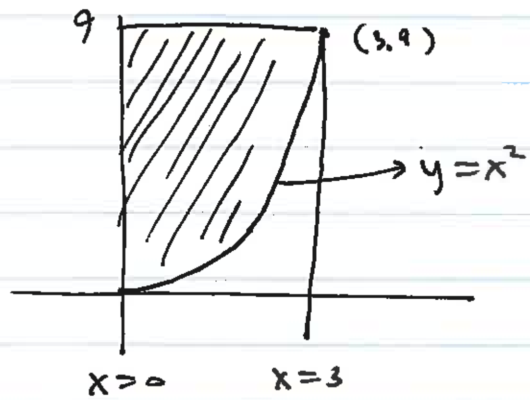


Ans:



Pract. #3 $\displaystyle\int_0^3 \int_{x^2}^9 4x e^{y^2} \, dy \, dx$

I $\left\{ \begin{array}{l} 0 \le x \le 3 \\ x^2 \le y \le 9. \end{array} \right.$



II $\left\{ \begin{array}{l} 0 \le y \le 9 \\ 0 \le x \le \sqrt{y} \end{array} \right.$

$$\int_0^9 \int_0^{\sqrt{y}} 4x e^{y^2} \, dx \, dy = \int_0^9 2x^2 e^{y^2} \Big|_{x=0}^{x=\sqrt{y}}$$

$$= \int_0^9 \left(2(\sqrt{y})^2 e^{y^2} - 0 \right) dy$$

$$= \int_0^9 2y e^{y^2} \, dy = \int_0^{81} e^u \, du$$

$u = y^2$
$du = 2y\,dy$

$$= e^u \Big|_0^{81} = e^{81} - 1.$$

Practice #5

$$X(t) = (4\cos t, 4\sin t, 3t) \qquad 0 \le t \le 2\pi$$

a) Velocity $X'(t) = (-4\sin t, 4\cos t, 3)$

Acceleration $X''(t) = (-4\cos t, -4\sin t, 0)$

speed $|X'(t)| = \sqrt{16\sin^2 t + 16\cos^2 t + 9} = \sqrt{25} = 5$

b) Tangent line $\ell(s) = X(a) + s\,X'(a)$

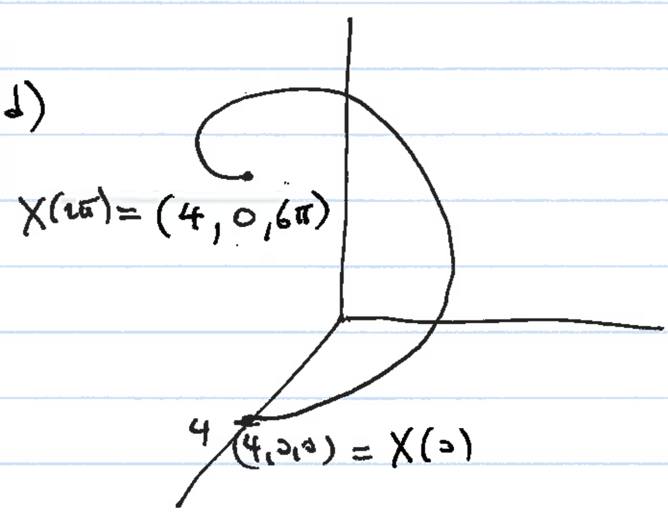
$X(\pi) = (-4, 0, 3\pi)$

$X'(\pi) = (0, -4, 3)$

$$\ell(s) = (-4, 0, 3\pi) + s(0, -4, 3)$$

c) $$\int_0^{2\pi} |X'(t)|\,dt = \int_0^{2\pi} 5\,dt = 10\pi.$$

d)



Practice #8

$h = x^2 - y^2 - 2y$

a) $\nabla h = (2x, -2y-2)$

$$\nabla h = 0 \iff \left.\begin{array}{r} 2x = 0 \\ -2y-2 = 0 \end{array}\right\} \iff \begin{array}{l} x = 0 \\ y = -1 \end{array}$$

$(0,-1)$ c.p.

$$H_f = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \qquad \det = -4$$

Saddle

b) Lagrange multiplers

$g = x^2 + y^2 = 1$

$\nabla g = (2x, 2y)$

① $2x = 2\lambda x$

② $-2y - 2 = 2\lambda y$

③ $x^2 + y^2 = 1$

$2x - 2\lambda x = 0$

$2x(1-\lambda) = 0$

$x = 0$ or. $\lambda = 1$.

③ $y = \pm 1$

② $-2y - 2 = 2y$

$-2 = 4y$

$y = -\frac{1}{2}$

$1 = x^2 + y^2 = x^2 + \frac{1}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

	$x^2 - y^2 - 2y$
$(0,1)$	$-1 - 2 = -3$
$(0,-1)$	$-1 + 2 = 1$
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$\frac{3}{4} - \frac{1}{4} + 1 = \frac{3}{2}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$\frac{3}{4} - \frac{1}{4} + 1 = \frac{3}{2}$

$\frac{3}{2}$ max value

-3 min. value

Practice #9 $f = x^2y^3 - yz + 2xz$

a) $$\nabla f = (2xy^3 + 2z,\ 3x^2y^2 - z\ ,\ -y + 2x)$$

$$\nabla f(1,0,2) = (4, -2, 2)$$

b) $$D_u f(1,0,2) = \nabla f(1,0,2) \cdot u = (4,-2,2)\frac{(2,-2,1)}{3} = \frac{14}{3}$$

$$u = \frac{(2,-2,1)}{\sqrt{4+4+1}} = \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right) \text{ unit}$$

c) f increases fastest in the direction $\dfrac{(4,-2,2)}{\sqrt{24}}$.

d) Tangent plane $x^2y^3 - yz + 2xz = 4$

at $(1,0,2)$

$$\nabla f(a) \cdot (\bar{x} - \bar{a}) = 0$$

$$(4, -2, 2) \cdot (x-1, y-0, z-2) = 0$$

$$4x - 2y + 2z = (1,0,2) \cdot (4,-2,2)$$
$$4x - 2y + 2z = 8$$

Practice 12a

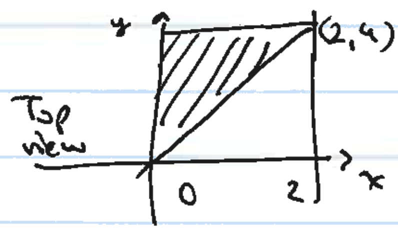
$$\int_a^b dx \int_c^d dy \int_e^f dz \; f = \int_a^b \int_c^d \int_e^f F \; dz\, dy\, dx$$

$$\int_0^2 dx \int_{2x}^4 dy \int_0^y e^{z} dz$$

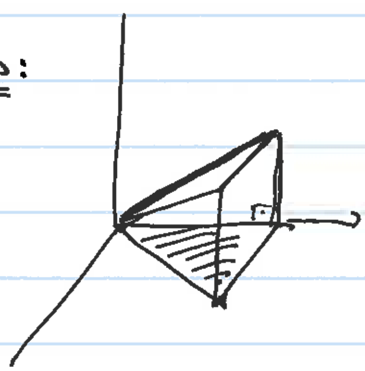
$$\left.\begin{array}{l} 0 \le x \le 2 \\ 2x \le y \le 4 \end{array}\right\} \longrightarrow$$

view from z axis direction

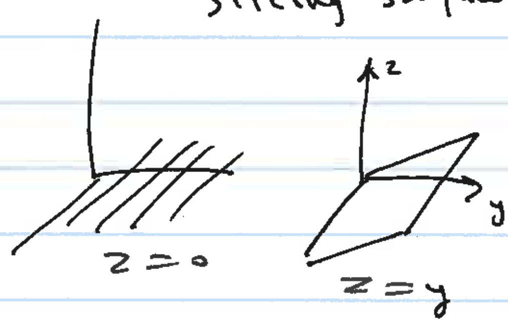
$0 \le z \le y$



Ans:



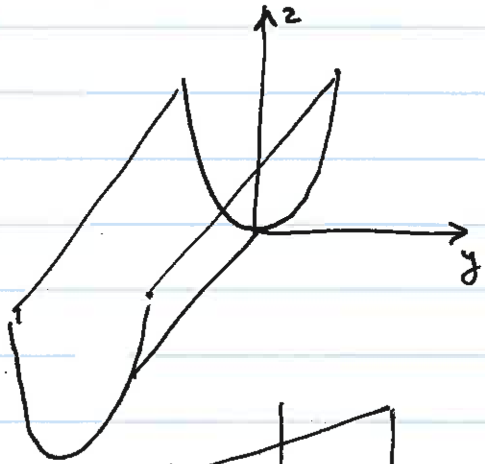
slicing surfaces



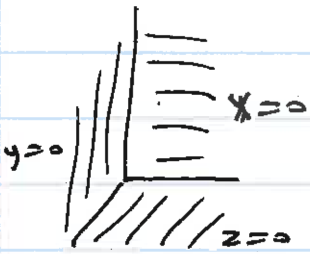
pract #13 $y^2 \geq z$

$x + y \leq 1$

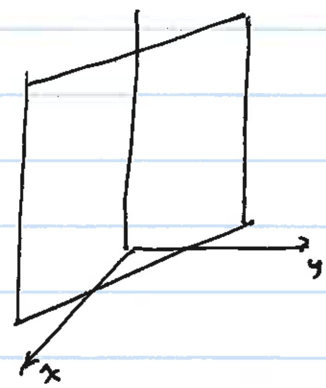
$x, y, z \geq 0$



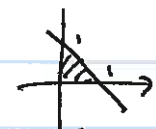
$y^2 = z$

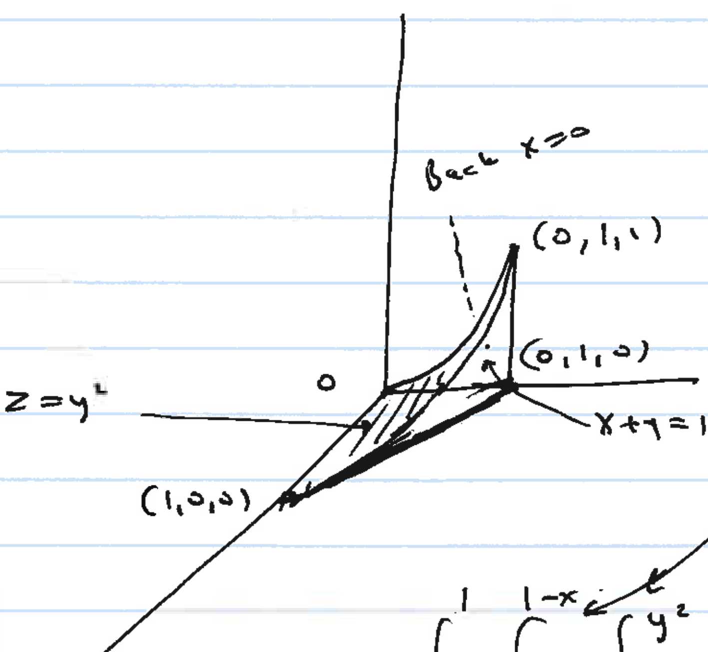


slicing surfaces



$x + y = 1$





$$\int_0^1 \int_0^{1-x} \int_0^{y^2} 12yz \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} 6yz^2 \Big|_{z=0}^{z=y^2} dy \, dx = \int_0^1 \int_0^{1-x} 6y^5 \, dy \, dx$$

practice #13
Continue

c) $$\int_0^1 y^6 \Big|_0^{1-x} dx = \int_0^1 (1-x)^6 dx = \int_1^0 -u^6 du$$

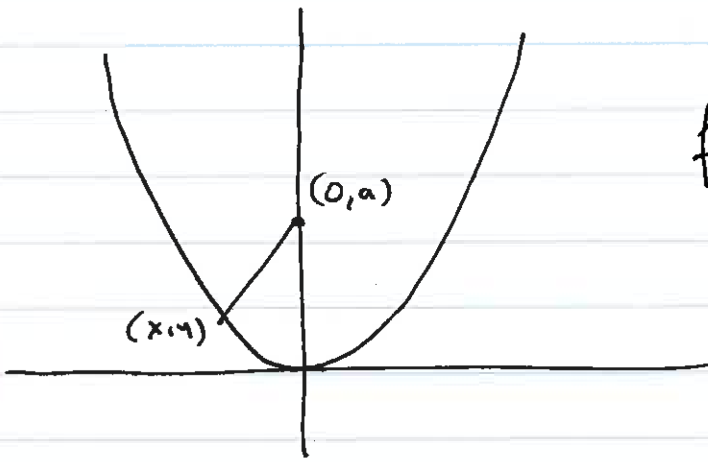
$u = 1-x$
$du = -dx$

$$= \int_0^1 u^6 du = \frac{1}{7}.$$

b) $$\int_0^1 \int_0^{1-x} \int_0^{y^2} 1 \, dz\, dy\, dx = \text{volume}.$$

Practice #17 $S = \{(x,y) \mid 2y = x^2\}$

$$y = \frac{1}{2}x^2$$



$f = (x-0)^2 + (y-a)^2$ min

$= x^2 + (y-a)^2.$

$g = x^2 - 2y$ constraint

L. Multip. $\nabla f = \lambda \nabla g$

$\nabla f = (2x, 2(y-a))$

$\nabla g = (2x, -2)$

solve: $\begin{cases} 2x = 2\lambda x & (1) \\ 2(y-a) = -2\lambda & (2) \\ x^2 = 2y & (3) \end{cases}$

$2x - 2\lambda x = 0$

$2x(1-\lambda) = 0$

$x = 0$ $\lambda = 1.$

(3) $y = 0$ $2(y-a) = -2$

$y - a = -1$

$\underbrace{0 \leq y}_{} = a - 1$

If $a < 1$, no sol^{n}.

If $a \geq 1$ $y = a - 1$ sol^{n}.

#17 Continue

Case 1 $\quad a < 1$

	$x^2+(y-a)^2 = \text{dist}^2$
(0,0)	a^2

closest pt.

Case 2 $\quad a \geqslant 1$

$y = a-1$

$2y = x^2$

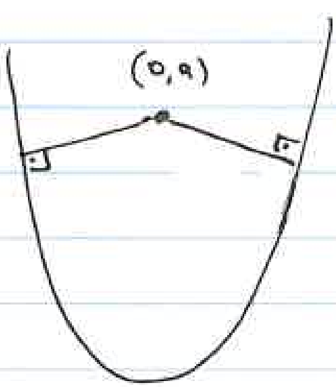
$x^2 = 2(a-1)$

$x = \pm\sqrt{2(a-1)}$

	$x^2+(y-a)^2$
$(0,0)$	a^2
$(\sqrt{2(a-1)}, a-1)$	$2(a-1)+1 = 2a-1$
$(-\sqrt{2(a-1)}, a-1)$	$= 2a-1$

smaller than a^2 if $a<1$.

closest pts



$(a-1)^2 \geqslant 0$

$a^2-2a+1 \geqslant 0$

$a^2 \geqslant 2a-1$

p277 #38

$$f = x^2 y$$

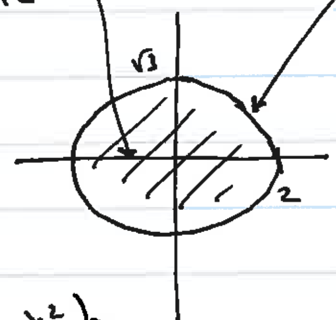
$$D = \{(x,y) \mid 3x^2 + 4y^2 \le 12\}$$

Interior

$$3x^2 + 4y^2 < 12$$

Boundary

$$3x^2 + 4y^2 = 12$$

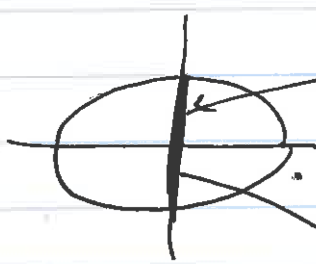


$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\nabla f = (2xy, x^2)$$

$$Hf = \begin{bmatrix} 2y & 2x \\ 2x & 0 \end{bmatrix}$$

$$x = 0 \quad \left\{ \begin{array}{l} 2xy = 0 \\ x^2 = 0 \end{array} \right.$$



all degenerate c.p.

$f \equiv 0$ on this segment

#38 continue

Boundary $g = 3x^2 + 4y^2 = 12$

$f = x^2 y$ $\qquad \nabla g = (6x, 8y)$

$\nabla f = \lambda \nabla g$

$$2xy = \lambda \cdot 6x \quad ①$$
$$x^2 = \lambda 8y \quad ②$$
$$3x^2 + 4y^2 = 12 \quad ③$$

$$2xy - 6\lambda x = 0$$
$$2x(y - 3\lambda) = 0$$

$x = 0$

③ $4y^2 = 12$

$y^2 = 3$

$y = \pm\sqrt{3}$

$y = 3\lambda$

$\lambda = \frac{y}{3}$

② $x^2 = 8\lambda y = 8 \cdot \frac{y}{3} y = \frac{8}{3} y^2$

$x^2 = \frac{8}{3} y^2$

③ $3x^2 + 4y^2 = 12$

$8y^2 + 4y^2 = 12$

$12y^2 = 12$

$y = \pm 1$

$x^2 = \frac{8}{3}$

$x = \pm\frac{2\sqrt{2}}{\sqrt{3}}$

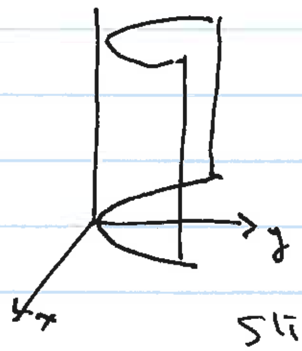
	$x^2 y$	
$(0, \sqrt{3})$	0	
$(0, -\sqrt{3})$	0	
$\left(\frac{2\sqrt{2}}{\sqrt{3}}, 1\right)$	$\frac{8}{3}$	max
$\left(\frac{2\sqrt{2}}{\sqrt{3}}, -1\right)$	$-\frac{8}{3}$	min
$\left(-\frac{2\sqrt{2}}{\sqrt{3}}, 1\right)$	$\frac{8}{3}$	
$\left(-\frac{2\sqrt{2}}{\sqrt{3}}, -1\right)$	$-\frac{8}{3}$	
$(0, a)$	0	

5.4 p348 #13

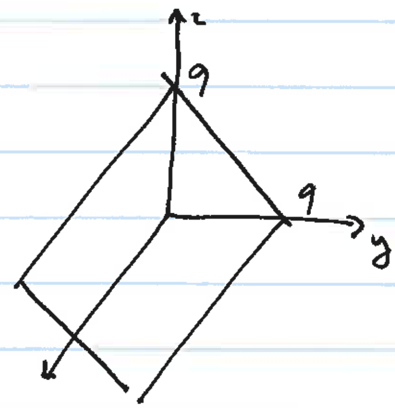
$y = x^2$

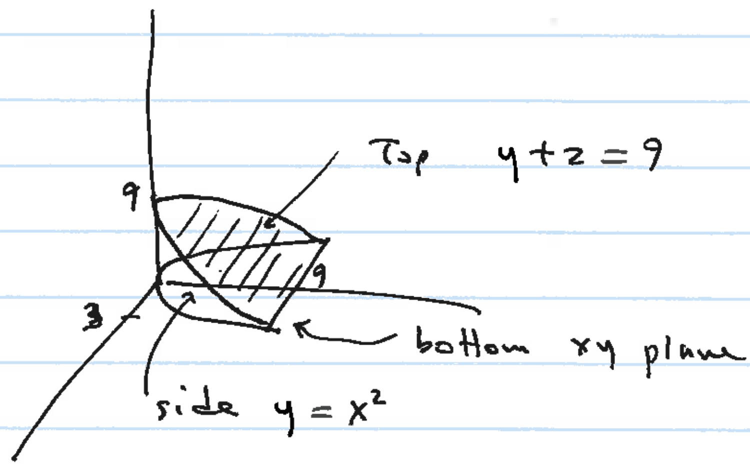
$y + z = 9$

& xy-plane

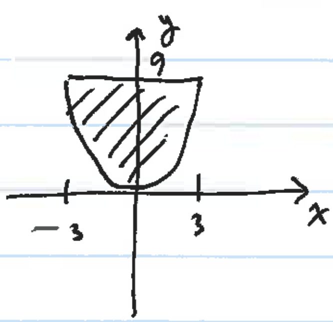


Slicing surfaces



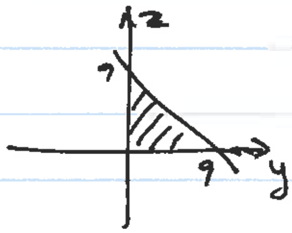


$$\int_{-3}^{3} \int_{x^2}^{9} \int_{0}^{z=9-y} 8xyz \, dz \, dy \, dx.$$



p348 #13
continue

$$\int_0^9 \int_0^{9-y} \int_{-\sqrt{y}}^{\sqrt{y}} \delta xyz \, dx \, dz \, dy$$



view from x-axis direction.

$$I = \int_0^9 \int_0^{9-y} \underbrace{\frac{\delta x^2}{2} yz \Bigg|_{x=-\sqrt{y}}^{x=\sqrt{y}}}_{0} = 0 .$$