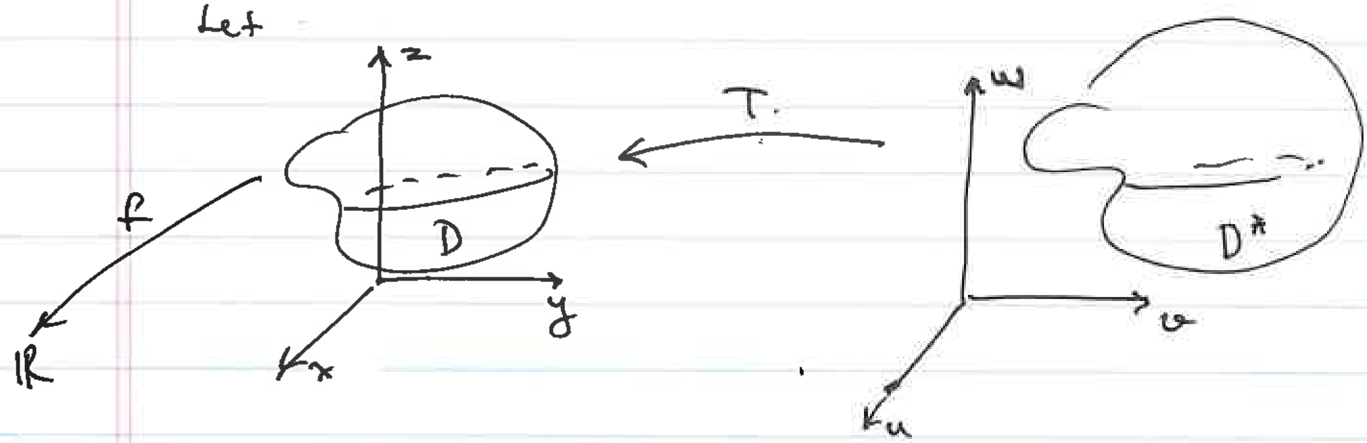
Announcement: 5.5 is not in the 2^{nd} MT.
It is in the final.

Thm: Jacobi $(n=3)$

Let



$T(u,v,w) = (x,y,z)$

T is a coordinate transformation from D^* onto D.

(1-1, continuously diff'ble, $\det DT \neq 0$)

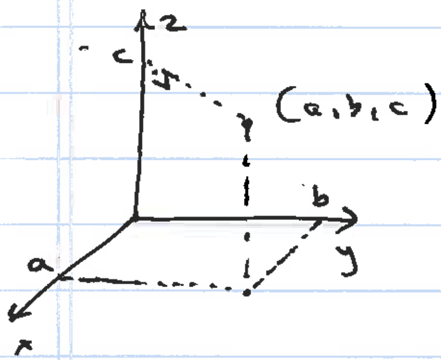
$f: D \to \mathbb{R}$ continuous: Then

$$\iiint_D f(x,y,z)\, dV_{xyz} = \iiint_{D^* = T^{-1}(D)} f(T(u,v,w)) \cdot |\det DT| \, dV_{uvw}$$

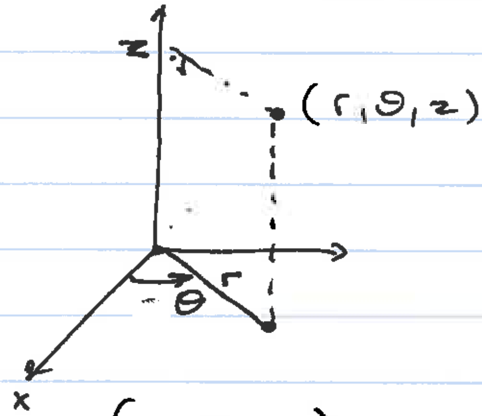
$dV_{xyz} = dx\,dy\,dz$ $\qquad$ $dV_{uvw} = du\,dv\,dw$

$$\det DT = \frac{\partial(x,y,z)}{\partial(u,v,w)}$$

CYLINDRICAL COORDINATES



Rectangular Coordinates

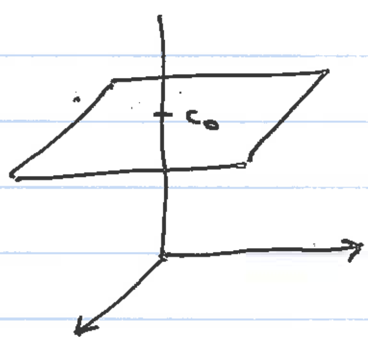


(r, θ, z) cylindrical coordinates

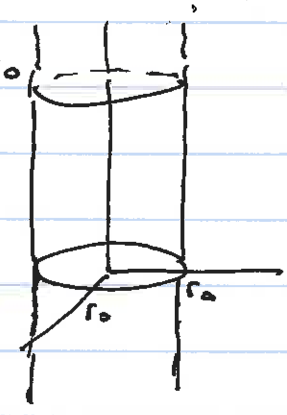
$$T \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$$T^{-1} \begin{cases} r = \sqrt{x^2+y^2} & r \geq 0 \\ \theta = \tan^{-1}\frac{y}{x} + k\pi & \theta \in [0, 2\pi]. \\ z = z & z \in \mathbb{R} \end{cases}$$

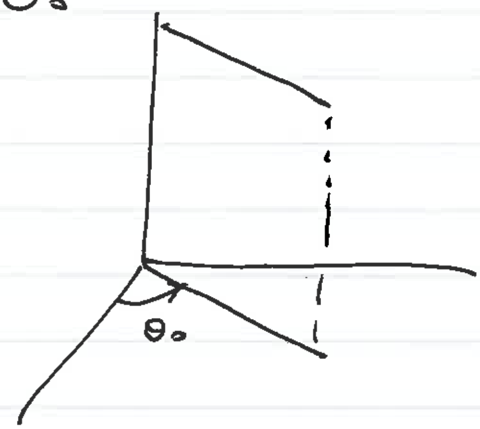
$z = c_0$



$r = r_0$



$\Theta = \Theta_0$



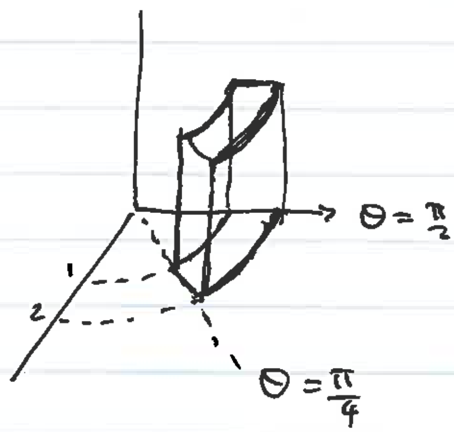
Half planes with z-axis as an edge.

Ex

$1 \leq r \leq 2$

$\frac{\pi}{4} \leq \Theta \leq \frac{\pi}{2}$

$0 \leq z \leq 1$



$|\det DT| = ?$

$$T \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad DT = \begin{bmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

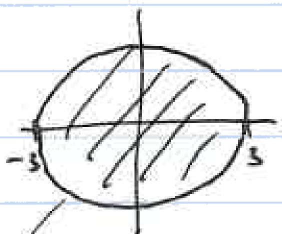
$$\det DT = r = \frac{\partial(x,y,z)}{\partial(r,\theta,z)}$$

$$\boxed{dx\,dy\,dz = r\,dr\,d\Theta\,dz}$$

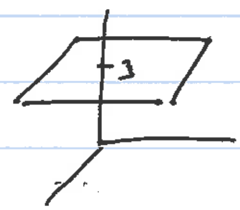
5.5 Exc # 28 p372

$$I = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^{3} \frac{e^z}{\sqrt{x^2+y^2}}\, dz\, dy\, dx.$$

$$\left.\begin{aligned} -3 &\le x \le 3 \\ -\sqrt{9-x^2} &\le y \le \sqrt{9-x^2} \end{aligned}\right\} \text{ gives top view}$$



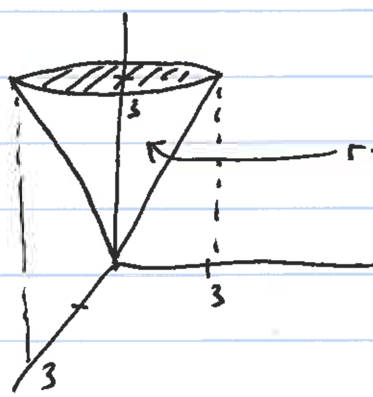
$z = 3$



$z = \sqrt{x^2+y^2}$

z bounds

D =



$r = z = \sqrt{x^2+y^2}$

$$\left\{\begin{aligned} 0 &\le r \le 3 \\ 0 &\le \theta \le 2\pi \end{aligned}\right.$$

$z = 3$ upper cap.

$z = r.$ bottom surface

$$r \le z \le 3$$

$0 \le r \le 3$
$0 \le \theta \le 2\pi$
$r \le z \le 3$

Ex28 Continue

$|\det T|$

$$I = \int_0^{2\pi} \int_0^3 \int_r^3 \frac{e^z}{\cancel{r}} \, \cancel{r} \; dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^3 e^z \Big|_{z=r}^{z=3} \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^3 (e^3 - e^r) \, dr \, d\theta$$

$$= \int_0^{2\pi} e^3 r - e^r \Big|_{r=0}^{r=3} \, d\theta$$

$$= \int_0^{2\pi} (3e^3 - e^3) - (0 - 1) \, d\theta$$

$$= 2\pi \cdot (3e^3 - e^3 + 1).$$

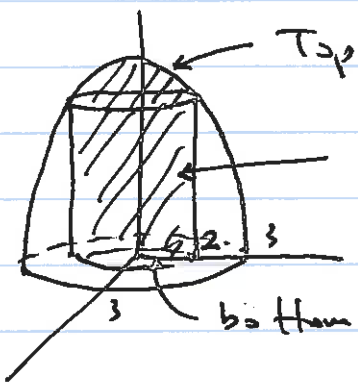
5.5 #40 p 373

Volume of solid W bdd

paraboloid $z = 9 - x^2 - y^2$

cylinder $x^2 + y^2 = 4$

xy-plane



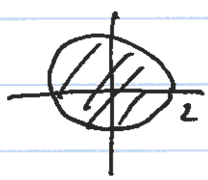
Top $z = 9 - x^2 - y^2$

side $x^2 + y^2 = 4$

bottom xy-plane

Look at it in cylindrical coordinates:

Top view



$0 \le r \le 2$

$0 \le \theta \le 2\pi$

Bottom $z = 0$

Top $z = 9 - r^2$ $\quad (r^2 = x^2 + y^2)$

$$\text{Volume} = \iiint_D 1 \, dx\,dy\,dz = \int_0^2 \int_0^{2\pi} \int_0^{9-r^2} 1 \cdot r \; dz \, d\theta \, dr$$

|det DT|

$$= \int_0^2 \int_0^{2\pi} rz \Big|_{z=0}^{9-r^2} d\theta\, dr$$

$$= \int_0^2 \int_0^{2\pi} \left(r(9-r^2) - 0 \right) d\theta\, dr$$

$$= 2\pi \int_0^2 (9r - r^3)\, dr$$

$$= 2\pi \left(\frac{9r^2}{2} - \frac{r^4}{4} \Big|_0^2 \right)$$

$$= 2\pi \left(\frac{9.4}{2} - \frac{16}{4} - 0 \right)$$

$$= 2\pi (18 - 4) = 28\pi .$$