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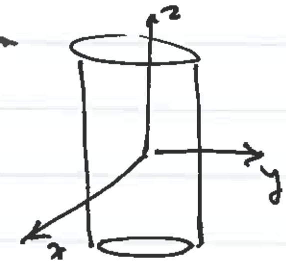
5.4 #24

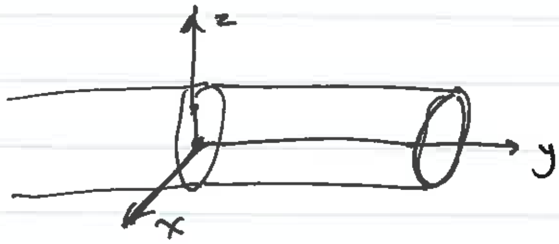
Take $a=1$

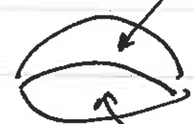
Volume of the Region Inside the cylinders

$x^2+y^2=1$

$x^2+z^2=1$



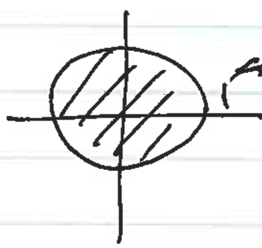


Round ball $\neq$  Football

cylindrical

cylindrical pieces

Volume?

Top view 

$-1 \leq x \leq 1$

$-\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}$

surface bounding above $x^2+z^2=1$

" " " $x^2+z^2=1$

$z=\sqrt{1-x^2}$ $\quad z=-\sqrt{1-x^2}$

(A) $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1 \, dz\, dy\, dx.$$

B $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1\,dz\,dx\,dy$ Harder integral

We'll Do A Easier of the two.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 1\,dz\,dy\,dx$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} z\Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2\sqrt{1-x^2}\,dy\,dx$$

$$= \int_{-1}^{1} 2y\sqrt{1-x^2}\Big|_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= \int_{-1}^{1} 4(1-x^2)\,dx$$

$$= 4\cdot\left(x-\frac{x^3}{3}\Big|_{-1}^{1}\right) = 4\cdot\left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)=\frac{16}{3}$$

For $a \neq 1$

Answer is

$\frac{16}{3}a^3 = V.$

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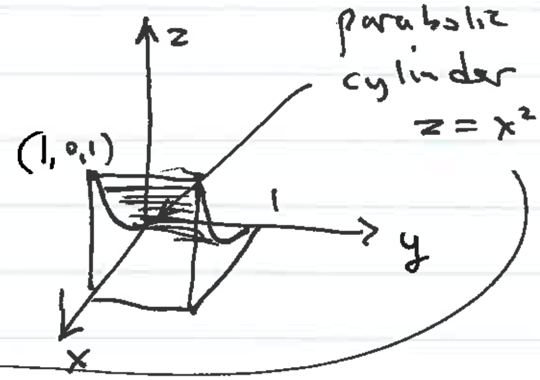
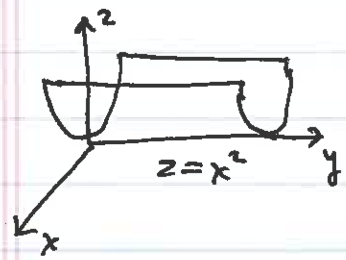
$$\int_0^1 \int_0^1 \int_0^{x^2} f \, dz \, dy \, dx.$$

$0 \le x \le 1$
$0 \le y \le 1$
$0 \le z \le x^2$

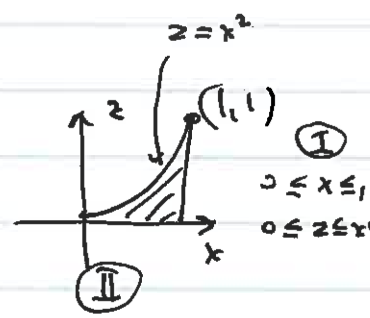




$$\int_0^1 \int_0^1 \int_0^{x^2} f \, dz \, dx \, dy$$

$$\int_0^1 \int_0^{x^2} \int_0^1 f \, dy \, dz \, dx$$

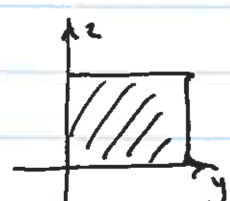
$$\int_0^1 \int_{\sqrt{z}}^1 \int_0^1 f \, dy \, dx \, dz$$



$0 \le z \le 1$
$\sqrt{z} \le x \le 1$

$$\int_0^1 \int_0^1 \int_{\sqrt{z}}^1 f \, dx \, dy \, dz$$

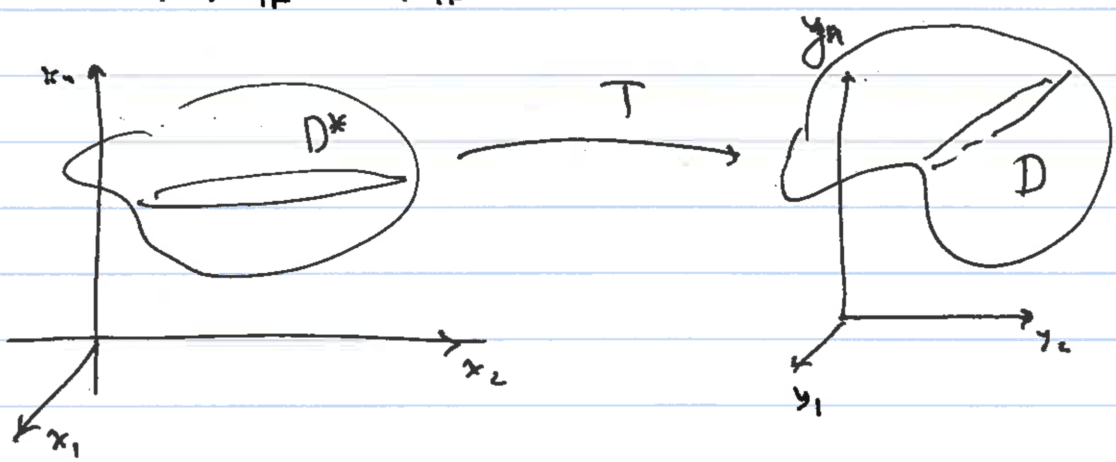
$$\int_0^1 \int_0^1 \int_{\sqrt{z}}^1 f \, dx \, dz \, dy$$



5.5 Change of Variables / SUBSTITUTION

I Coordinate transformations.

$$T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$



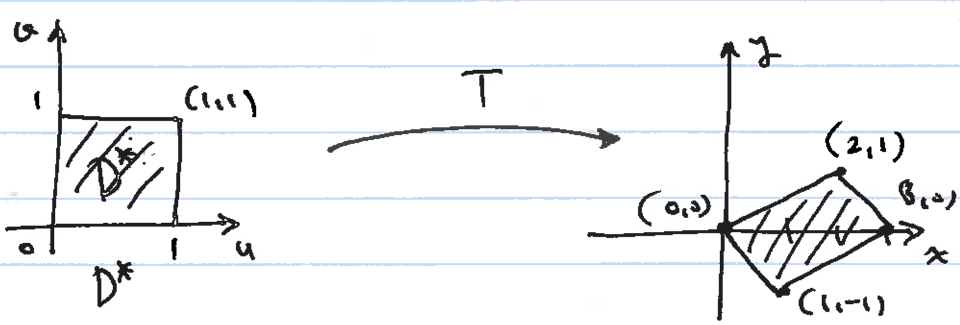
Want : T must be 1-1

$\det(DT) = \det(T') \neq 0$

DT continuous

Ex 1. $T(u,v) = (2u+v,\ u-v) : \mathbb{R}^2 \to \mathbb{R}^2$

Linear maps take lines to lines



$D = T(D^*)$

$$T(0,0) = (0,0)$$
$$T(1,0) = (2,1)$$
$$T(0,1) = (1,-1)$$
$$T(1,1) = (3,0)$$

$$DT = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

$\det DT = -3$ ← Area is 3 times larger

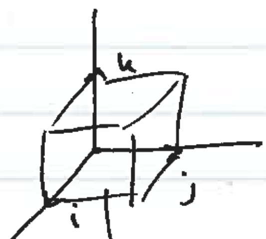
flipped

Thm: $L : \mathbb{R}^n \to \mathbb{R}^n$ Linear map.

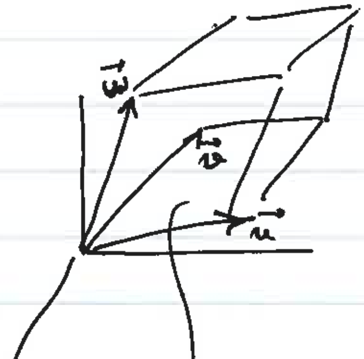
$$\text{volume}_n (L(D^*)) = \underbrace{|\det DL|}_{\text{volume/area stretching factor}} \cdot \text{volume}_n (D^*)$$

Ex 2 $L : \mathbb{R}^3 \to \mathbb{R}^3$

$$L(1,0,0) = \vec{u}$$
$$L(0,1,0) = \vec{v}$$
$$L(0,0,1) = \vec{w}$$



volume = 1



volume of parallelepiped

$$= |(\vec{u} \times \vec{v}) \cdot \vec{w}|$$

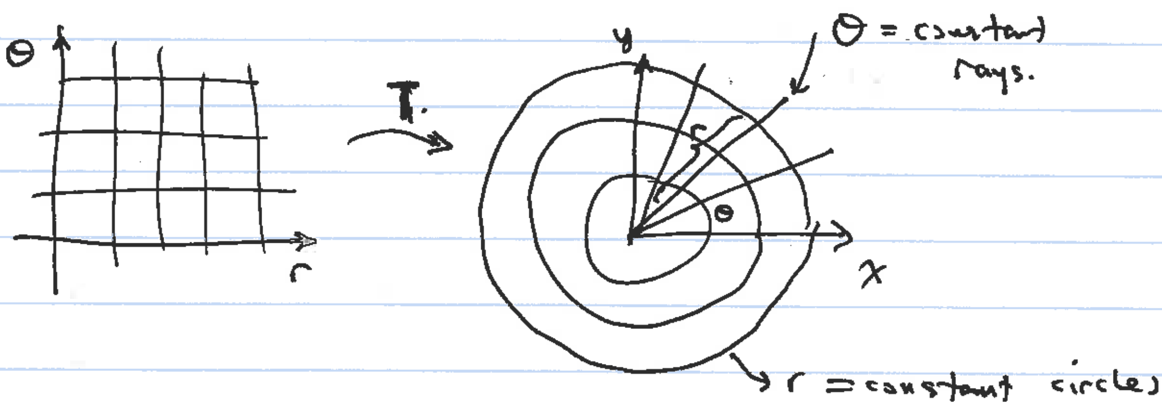
$$DL = \begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix}$$

$$= \left| \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \right|$$

$$= |\det (DL)^T|$$

Polar Coordinates

Ex 3. $T(r,\theta) = (r\cos\theta, r\sin\theta) = (x,y)$



$$DT = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$\det DT = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta - (-r\sin^2\theta)$$

$$= r.$$

same r.

Recall Calc II: $dx\,dy = r\,dr\,d\theta$