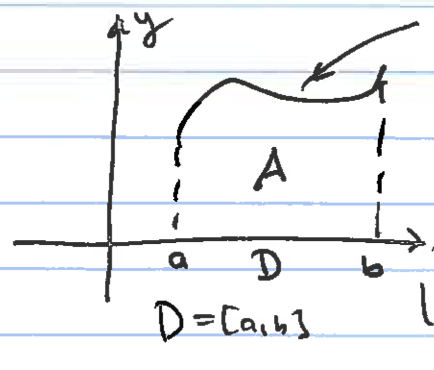
Oct 28, 2016

1

5.4

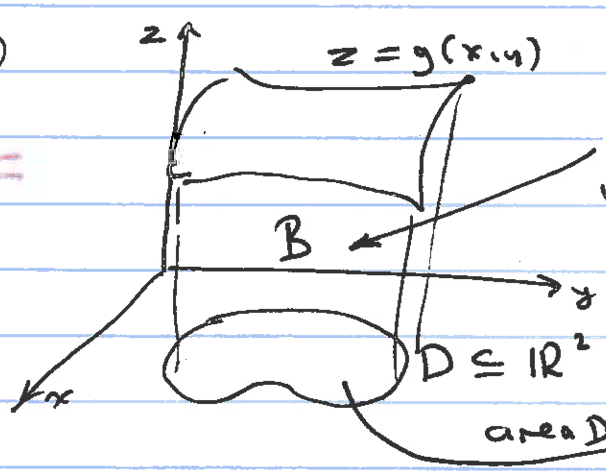
n=1



$$\text{Area } A = \int_a^b f(x)\,dx$$

$$\text{length of } D = \int_a^b 1\,dx = b-a$$

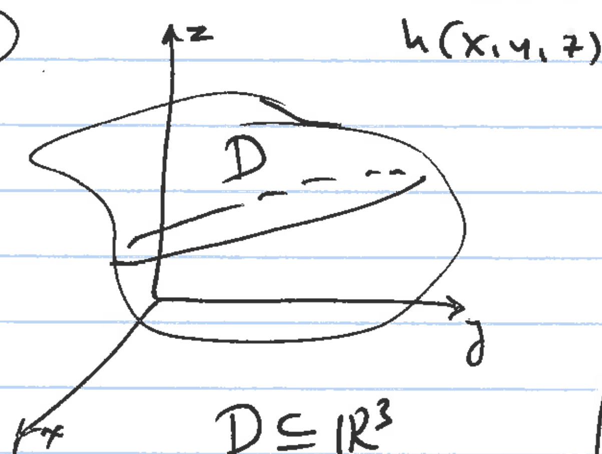
n=2



$$\text{volume of } B = \iint_D g(x,y)\,dA$$

$$\text{area } D = \iint_D 1\,dA$$

n=3



$$\iiint_D h(x,y,z)\,dV$$

$$\iiint_D 1\,dV = \text{volume } D$$

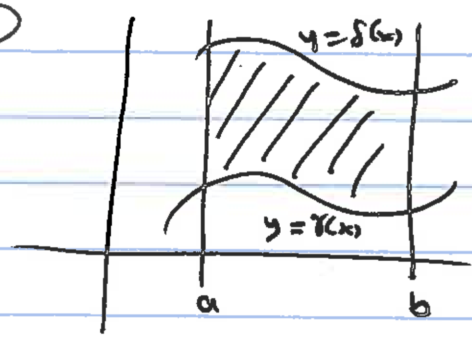
If $h(x,y,z)$ is the density at (x,y,z) of an object filling D, then $\iiint_D h(x,y,z)\,dV$ will be the mass of the object.

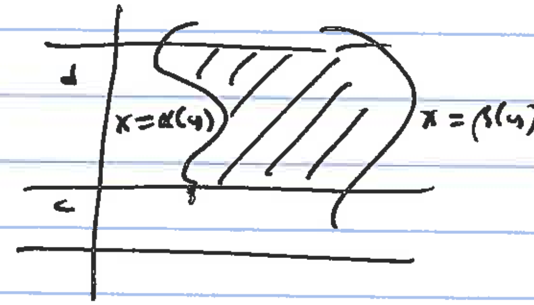
Elementary Regions

I

II

$n=2$





$a \le x \le b$

$\gamma(x) \le y \le \delta(x)$

$c \le y \le d$

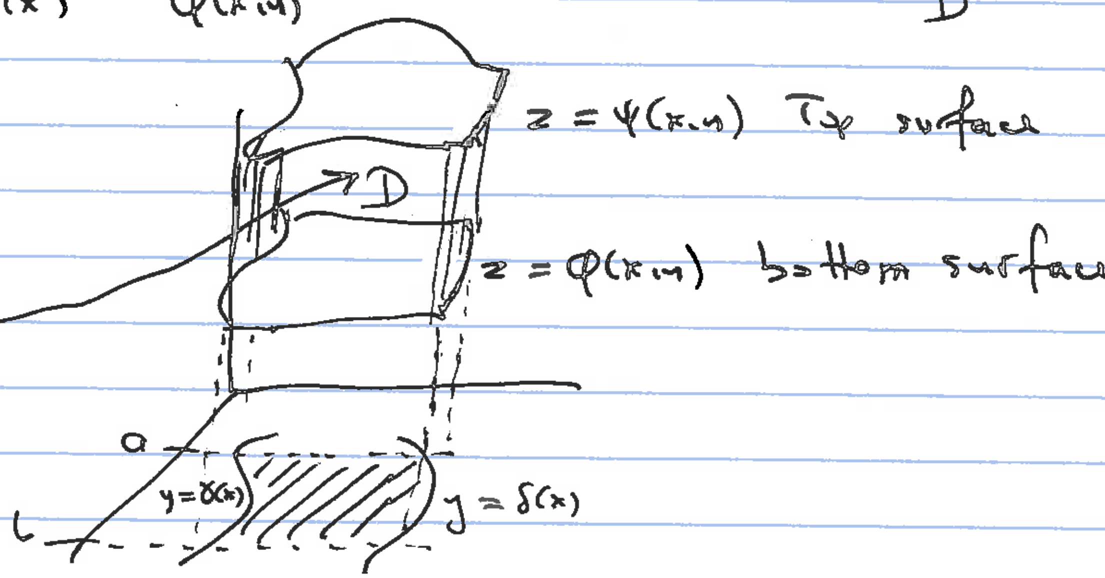
$\alpha(y) \le x \le \beta(y)$

$n=3$ Elementary Regions:

$$D \left\{ \begin{aligned} & a \le x \le b \\ & \gamma(x) \le y \le \delta(x) \\ & \varphi(x,y) \le z \le \psi(x,y) \end{aligned} \right\}$$

There are 6 ways to create Elementary regions in $\mathbb{R}^3$

$$\int_a^b \int_{\gamma(x)}^{\delta(x)} \int_{\varphi(x,y)}^{\psi(x,y)} g(x,y,z)\, dz\, dy\, dx = \iiint_D g(x,y,z)\, dV$$



p348 Exc #6

$$\int_1^3 \int_0^z \int_1^{xz} (x + 2y + z)\, dy\, dx\, dz.$$

$$= \int_1^3 \int_0^z xy + y^2 + zy \Big|_{y=1}^{y=xz} dx\, dz.$$

$$= \int_1^3 \int_0^z (x^2 z + x^2 z^2 + xz^2) - (x + 1 + z)\, dx\, dz$$

$$= \int_1^3 \frac{x^3 z}{3} + \frac{x^3 z^2}{3} + \frac{x^2 z^2}{2} - \frac{x^2}{2} - x - xz \Big|_{x=0}^{x=z} dz$$

$$= \int_1^3 \left(\frac{z^4}{3} + \frac{z^5}{3} + \frac{z^4}{2} - \frac{z^2}{2} - z - z^2 - 0 \right) dz$$

$$= \frac{z^5}{15} + \frac{z^6}{18} + \frac{z^5}{10} - \frac{z^3}{6} - \frac{z^2}{2} - \frac{z^3}{3} \Big|_{z=1}^{z=3}$$

$$= \left(\frac{3^5}{15} + \frac{3^6}{18} + \frac{3^5}{10} - \frac{3^3}{6} - \frac{3^2}{2} - \frac{3^3}{3} \right) - \left(\frac{1}{15} + \frac{1}{18} + \frac{1}{10} - \frac{1}{6} - \frac{1}{2} - \frac{1}{3} \right).$$

Ex
$$\int_0^2 \int_0^{2y} \int_{xy}^{3x+y} 2xy^2 z \, dz\, dx\, dy$$

$$= \int_0^2 \int_0^{2y} xy^2 z^2 \Big|_{z=xy}^{z=3x+y} dx\, dy$$

$$= \int_0^2 \int_0^{2y} xy^2 \Big[\underbrace{(3x+y)^2 - (xy)^2}_{9x^2+6xy+y^2-x^2y^2} \Big] dx\, dy$$

$$= \int_0^2 \int_0^{2y} \left(9x^3y^2 + 6x^2y^3 + xy^4 - x^3y^4 \right) dx\, dy$$

$$= \int_0^2 \frac{9x^4y^2}{4} + 2x^3y^3 + \frac{x^2y^4}{2} - \frac{x^4y^4}{4} \Big|_{x=0}^{x=2y} dy$$

$$= \int_0^2 \frac{9(2y)^4 \cdot y^2}{4} + \frac{2(2y)^3 y^3}{1} + \frac{(2y)^2 y^4}{2} - \frac{(2y)^4 \cdot y^4}{4} \, dy$$

$$= \int_0^2 \frac{144}{4} \cdot y^6 + 16y^6 + \frac{4}{2} y^6 - 4y^8 \, dy$$

$$= \int_0^2 54y^6 - 4y^8 \, dy = \frac{54}{7} y^7 - \frac{4}{9} y^9 \Big|_0^2 = \frac{54 \cdot 128}{7} - \frac{4 \cdot 512}{9} = \frac{47872}{63}$$