Oct 26, 2016

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ANNOUNCEMENTS

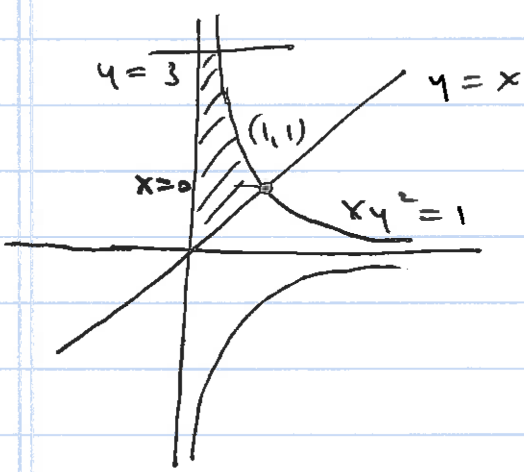
Extra office Hrs Wed 1:30 - 2:30
Except Nov 16

Quiz #4 Oct 28 Friday
4.1
4.2

Hw #9 due Monday 10/31 Monday

5.2 Exc #20 p 333

$$\iint_D 3y\,dA \qquad D \text{ bdd by } \begin{cases} xy^2 = 1 \\ y = x \\ x = 0 \\ y = 3. \end{cases}$$

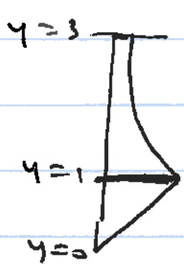


common sol^n

$x^3 = 1$

$x = 1$

Type II:



$$\int_{y=0}^{y=1} \int_{x=0}^{x=y} 3y \, dx\, dy \; +$$

$$\int_{y=1}^{y=3} \int_{x=0}^{x=1/y^2} 3y \, dx\, dy$$

$$\int_0^1 \int_0^y 3y \, dx \, dy + \int_1^3 \int_0^{1/y^2} 3y \, dx \, dy$$

$$= \int_0^1 3xy \Big|_{x=0}^{x=y} dy + \int_1^3 3xy \Big|_{x=0}^{x=1/y^2} dy$$

$$= \int_0^1 (3y^2 - 0) \, dy + \int_1^3 \left(3 \cdot \frac{1}{y^2} y - 0\right) dy$$

$$= y^3 \Big|_{y=0}^{y=1} + 3 \ln y \Big|_{y=1}^{y=3}$$

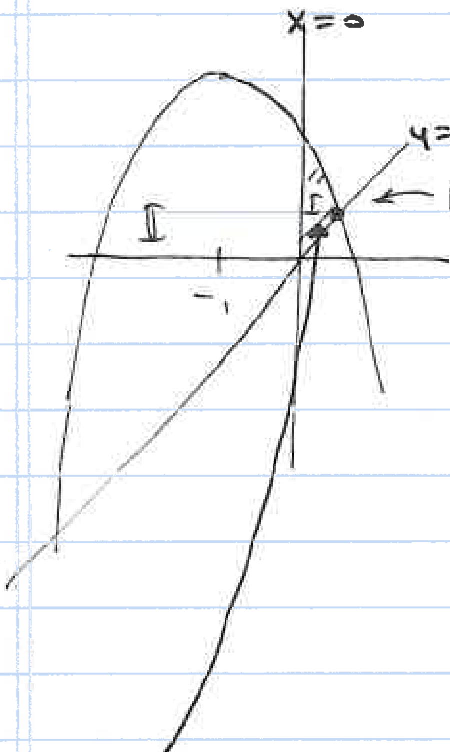
$$= (1^3 - 0^3) + 3(\ln 3 - \ln 1)$$

$$= 1 + 3 \ln 3$$

5.2 #28 p 333

Book didn't specify region I or II (it needs to)

Area of D bdd by $\begin{cases} y = 2x \\ x = 0 \\ y = 1-2x-x^2 \end{cases}$



So:

+ One needs to choose $x>0$ (or $x<0$)

← pt of intersection

$$1-2x-x^2 = 2x$$

$$0 = x^2+4x-1$$

$$x = \frac{-4 \pm \sqrt{16+4}}{2}$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

$$= -2 \pm \sqrt{5}$$

We choose: I^{st} quadrant $x>0$ $x = \sqrt{5}-2$

Area $= \iint_D 1\, dA$

$$\int_0^{\sqrt{5}-2} \int_{2x}^{1-2x-x^2} 1 \, dy\, dx = \int_0^{\sqrt{5}-2} y \Big|_{y=2x}^{y=1-2x-x^2} dx$$

Similar to how it is done in Calc I.

$$= \int_0^{\sqrt{5}-2} (1-2x-x^2) - 2x \, dx$$

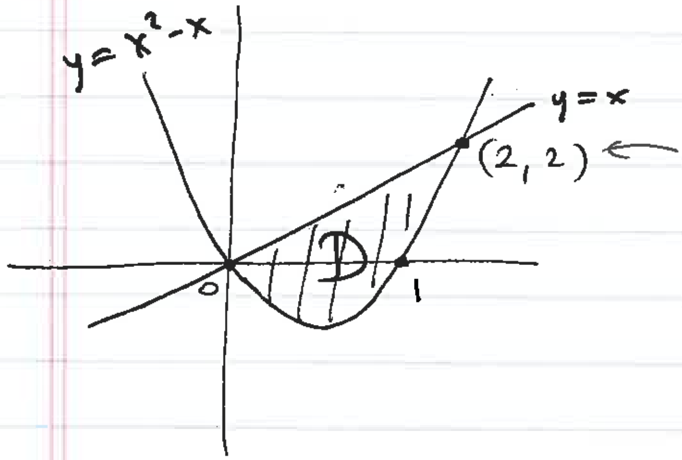
$$= \int_0^{\sqrt{5}-2} (1-4x-x^2)\, dx$$

$$= x - 2x^2 - \frac{x^3}{3} \Big|_0^{\sqrt{5}-2} = (\sqrt{5}-2) - 2(\sqrt{5}-2)^2 - \frac{(\sqrt{5}-2)^3}{3}$$

$$\cdots\cdots = 1/3\,(10\sqrt{5} - 22)$$

5.2 Ex # 36

Volume of the region D under $z = x^2 + 6y^2$ over the region D bdd by $y = x$, $y = x^2 - x$.



$$x = x^2 - x$$
$$0 = x^2 - 2x$$
$$x = 0, 2$$

$$V = \int_0^2 \int_{x^2-x}^{x} (x^2 + 6y^2)\, dy\, dx$$

$$= \int_0^2 x^2 y + 2y^3 \Big|_{y=x^2-x}^{x} dx$$

$$= \int_0^2 \Big((x^3 + 2x^3) - \big(x^2(x^2 - x) + 2(x^2-x)^3 \big) \Big) dx$$

$$\cdots\cdots = \int_0^2 (-2x^6 + 6x^5 - 7x^4 + 6x^3)\, dx$$

$$= -\frac{2}{7}x^7 + x^6 - \frac{7}{5}x^5 + \frac{6}{4}x^4 \Big|_0^2 = \cdots = \frac{232}{35}$$

5.3 Change the order of Integration

Easy:

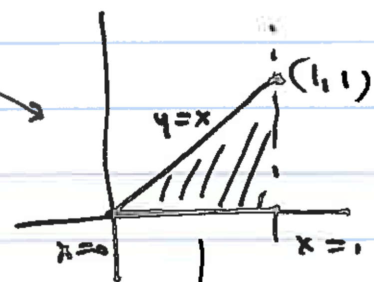
$a, b, c, d \in \mathbb{R}$

$$\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Only in this case, one can simply switch the order of the bounds. Not in any other case:

5.3 #2 $\text{I} = \int_0^1 \int_0^x (2-x-y)\,dy\,dx.$

Type I $\begin{cases} 0 \le x \le 1 \\ 0 \le y \le x \end{cases}$



$\left.\begin{array}{l} 0 \le y \le 1 \\ y \le x \le 1 \end{array}\right\}$ Type II

Correction:

$$\text{II} = \int_0^1 \int_{y}^{1} (2-x-y)\,dx\,dy$$

$\boxed{\underline{\text{Thm}}\ \text{I} = \text{II}.}$

We'll do: I

$$\int_0^1 \int_0^x (2-x-y)\,dy\,dx = \int_0^1 2y - xy - \frac{y^2}{2} \Big|_{y=0}^{y=x} dx$$

$$= \int_0^1 \left(\left(2x - x^2 - \frac{x^2}{2}\right) - 0\right) dx = \int_0^1 \left(2x - \frac{3x^2}{2}\right) dx = x^2 - \frac{x^3}{2}\Big|_0^1 = \frac{1}{2}$$

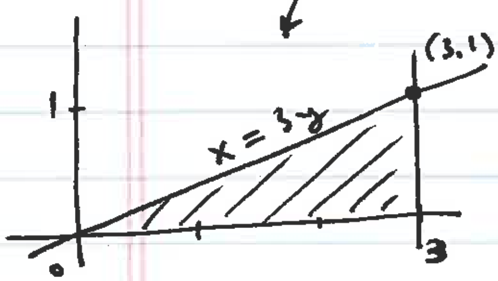
p 337

5.3 #14 $\int_0^1 \int_{3y}^3 \cos(x^2)\,dx\,dy$ <u>Evaluate!</u>

We have a problem:
$\int \cos(x^2)\,dx$ is not expressible as a finite sum of simple functions.

- <u>Change the order of integration:</u>

Type II $\begin{cases} 0 \le y \le 1 \\ 3y \le x \le 3 \end{cases}$



Type I

$0 \le x \le 3$
$0 \le y \le \frac{x}{3}$

$$\int_0^3 \int_0^{x/3} \cos(x^2)\,dy\,dx$$

$$= \int_0^3 y\cos(x^2)\Big|_{y=0}^{y=x/3} dx = \int_0^3 \frac{x}{3}\cos(x^2)\,dx$$

substitute $\begin{cases} u = x^2 \\ du = 2x\,dx \end{cases}$

$$\text{Integral} = \int_0^9 \frac{du}{6} \cos(u)\, du.$$

$$= \frac{1}{6} \sin u \Big|_0^9 = \frac{1}{6} \sin 9.$$