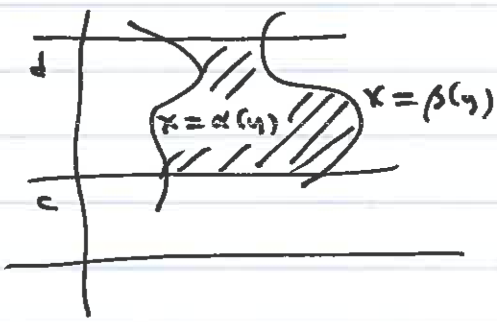
Oct 24, 2016

1

5.2 Continue.

Theorem: Let $f : D \to \mathbb{R}$ be continuous and D be of type II:

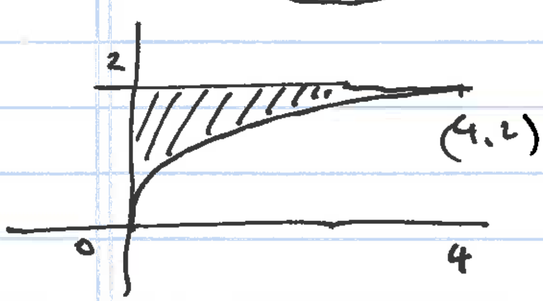
$$D = \{ (x,y) \mid c \le y \le d, \quad \alpha(y) \le x \le \beta(y) \}$$



Then
$$\iint_D f(x,y)\, dA = \int_c^d \left(\int_{\alpha(y)}^{\beta(y)} f(x,y)\, dx \right) dy$$

p 332 Exc 5.2 #5

$$\int_0^2 \int_0^{y^2} y\, dx\, dy = \int_0^2 \left(yx \Big|_{x=0}^{x=y^2} \right) dy$$



$0 \le y \le 2$

$0 \le x \le y^2$

$$= \int_0^2 (y^3 - 0)\, dy$$

$$= \frac{y^4}{4} \Big|_0^2 = \frac{(2^4 - 0^4)}{4} = 4.$$

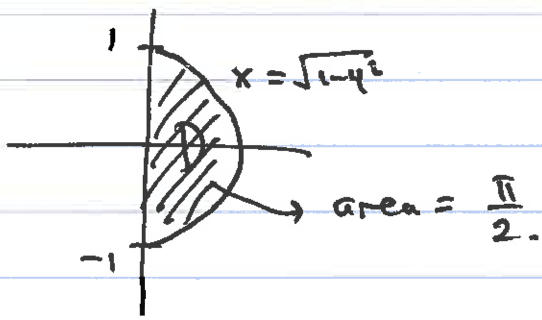
②

Exc #12

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} 3\,dx\,dy = \frac{3\pi}{2} = 3\cdot \text{Area}(D)$$

$$-1 \le y \le 1$$

$$0 \le x \le \sqrt{1-y^2}$$

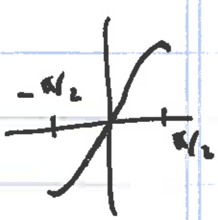


$$= \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} 3\,dx\,dy = \int_{-1}^{1} 3x \Big|_{x=0}^{x=\sqrt{1-y^2}} dy$$

$$= \int_{-1}^{1} 3\sqrt{1-y^2}\,dy = \int_{-\pi/2}^{\pi/2} 3\cos\theta\cos\theta\,d\theta.$$

$$y = \sin\theta \qquad dy = \cos\theta\,d\theta$$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$



$$I = \int_{-\pi/2}^{\pi/2} 3\cos^2\theta\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{1+\cos 2\theta}{2}\cdot 3\,d\theta$$

$$= 3\cdot\left(\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right)\Bigg|_{-\pi/2}^{\pi/2}$$

$$= 3\left[\left(\frac{\pi}{4} + \frac{\cancel{\sin \pi}^{\,0}}{4}\right) - \left(-\frac{\pi}{4} + \frac{\cancel{\sin -\pi}^{\,0}}{4}\right)\right]$$

$$= \frac{3\pi}{2}.$$

Thm: $\iint_D 1\, dA =$ area D, if LHS exists.

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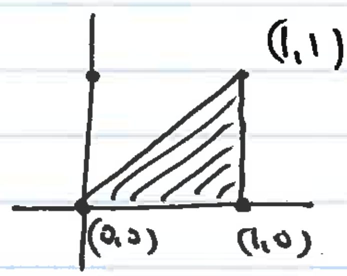
p 333 Exc #19

$$\iint_D e^{x^2} dA$$



$$\int_0^1 \underbrace{\int_0^x e^{x^2}\, dy}\, dx = \int_0^1 y e^{x^2}\Big|_{y=0}^{y=x} dx$$

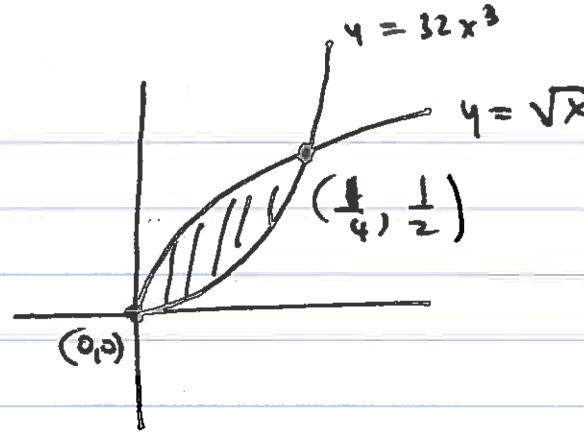
$$= \int_0^1 x e^{x^2} dx = \int_0^1 \frac{1}{2} e^u du = \frac{1}{2} e^u\Big|_{u=0}^{u=1} = \frac{1}{2}(e-1)$$

$x^2 = u$ $\quad 2x\,dx = du$

Exc #16 p 333

$f = 3xy$

Bounded by $\begin{cases} y = 32x^3 \\ y = \sqrt{x} \end{cases}$



$$32x^3 = \sqrt{x} \implies x = 0 \text{ OR } x = \frac{1}{4}$$

Since: if $x \neq 0$, $\frac{x^3}{\sqrt{x}} = \frac{1}{32}$

$$x^{5/2} = \frac{1}{32} = \frac{1}{2^5}$$

$$x^5 = \frac{1}{2^{10}} \qquad x = \frac{1}{4}$$

$$\int_0^{\frac{1}{4}} \int_{32x^3}^{\sqrt{x}} 3xy \, dy \, dx = \int_0^{\frac{1}{4}} \left(\frac{3xy^2}{2} \Big|_{y=32x^3}^{y=\sqrt{x}} \right) dx$$

$$= \int_0^{\frac{1}{4}} \left(\frac{3x^2}{2} - \frac{3x(32x^3)^2}{2} \right) dx$$

$3 \cdot 32^2 = 3(2^5)^2 = 3 \cdot 2^{10}$

$$= \int_0^{1/4} \left(\frac{3x^2}{2} - 1536x^7 \right) dx$$

$1536 = 512 \cdot 3$

$1536 = 2^9 \cdot 3$

$$= \frac{x^3}{2} - \frac{\cancel{1536}^{\,2^6 \cdot 3} \cdot x^8}{\cancel{8}_{\,1}} \Big|_0^{1/4} = \frac{x^3}{2} - 192x^8 \Big|_0^{1/4} = \frac{1}{128} - \frac{2^6 \cdot 3}{2^{16}}$$

$$= \frac{1}{128} - \frac{3}{2^{10}} = \frac{8 - 3}{1024} = \frac{5}{1024} \checkmark$$

$128 = 2^7$