Oct 20, 2016

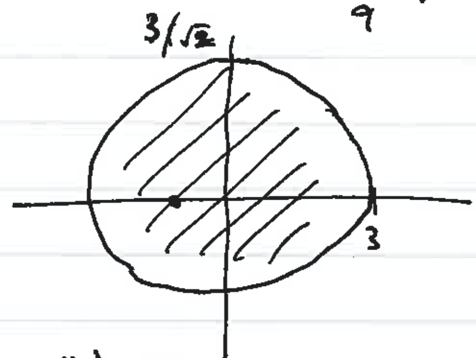
①

To Finish 4.3 & Chap IV:

Last Example:

Max/min $x^2 + 2x + 4y^2$ subject to $x^2 + 2y^2 \leq 9$.

$$\frac{x^2}{9} + \frac{2}{9}y^2 \leq 1$$



→ interior c.p. 1st, 2nd DT.

→ Boundary c.p. Lagrange Multipliers

Interior $f = x^2 + 2x + 4y^2$

$\nabla f = (2x+2, 8y)$

$\nabla f = 0 \iff x = -1$ & $y = 0$

$Hf = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \implies (-1,0)$ is a local min.

Boundary
Lagrange Mult. $\underbrace{x^2 + 2y^2}_{g} = 9$ (constraint), max/min $f = x^2 + 2x + 4y^2$

$\nabla g = (2x, 4y)$ $\nabla f = (2x+2, 8y)$

$\nabla f = \lambda \nabla g$

① $2x + 2 = 2\lambda x$

② $8y = 4\lambda y$

③ $x^2 + 2y^2 = 9$

② $0 = 8y - 4\lambda y = 4y(2-\lambda)$

OR

$y = 0$

③ $\Rightarrow$ $x = \pm 3$

Consistent with λ in ①

$\lambda = 2$.

① $2x + 2 = 4x$

$2 = 2x$

$x = 1$.

③ $1 + 2y^2 = 9$

$2y^2 = 8$

$y^2 = 4$

$y = \pm 2$

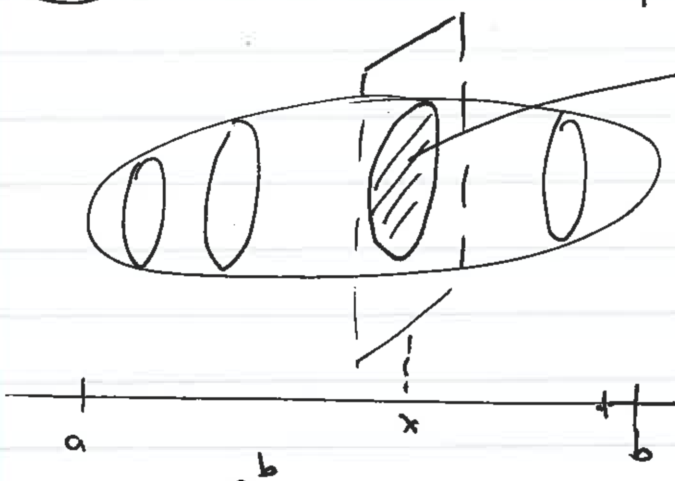
(c.p)	$x^2 + 2x + 4y^2$	
$(-1, 0)$	-1	← min value
$(3, 0)$	15	
$(-3, 0)$	3	
$(1, 2)$	19	max value.
$(1, -2)$	19	

Constraint set $\{(x,y) \mid x^2 + 2y^2 \leq 9\}$ is compact

END of Chap 4.

Chap 5

5.1 Cavalieri's Principle (Calc I)



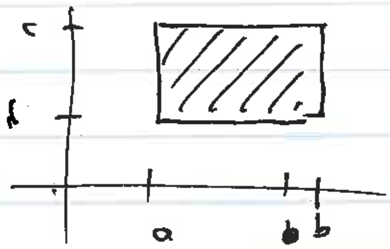
Area $= A(x)$ of the slice

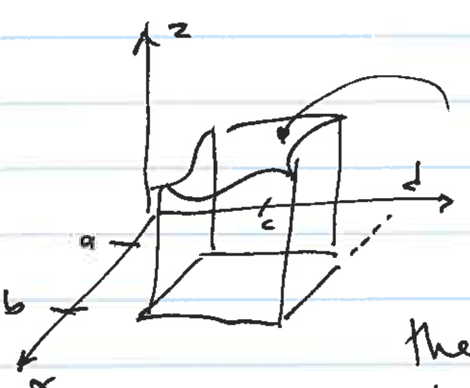
- Sliced by parallel planes
- Want Volume of the solid

$$\int_a^b A(x)\,dx = \text{Volume}.$$

$$f : [a,b]\times[c,d] \longrightarrow \mathbb{R}$$

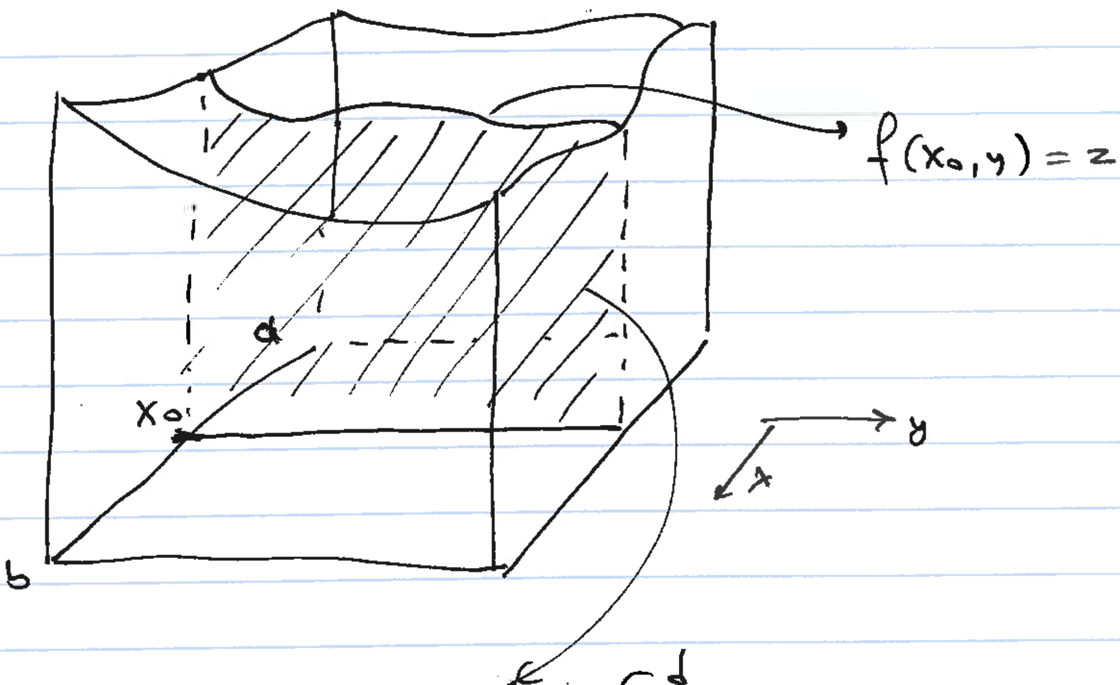
$\{(x,y) \mid a \le x \le b,\ c \le y \le d\}$





$z = f(x,y) \geq 0$

Want the volume V of the solid region bounded below by xy-plane; by the graph of $z = f(x,y)$ on top, and by the vertical planes $x=a$, $x=b$, $y=c$, $y=d$ on the 4 sides.



area of the slice: $A(x_0) = \int_c^d f(x_0,y)\,dy$

Thm: If f is continuous on $[a,b]\times[c,d]$:

$$V = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$$V = \int_c^d \left(\underbrace{\int_a^b f(x,y)\,dx} \right) dy.$$

$$\int_c^d \left(\underbrace{\int_a^b f(x,y)\,dx} \right) dy$$

→ First integrate w.r.t. x
assuming y's are constant
evaluate at $x = b$, $x = a$ } take difference

Next
→ You have a function of y, now
Integrate wrt y
evaluate at $y=c$, $y=d$ } take difference

Be Careful about which bounds are for which variables

$$\int_c^d \int_a^b f(x,y)\,dx\;dy$$

$$\int_c^d dy \int_a^b f(x,y)\,dx$$

Exc #2 p 313

$$\int_0^{\pi} \underbrace{\int_1^2 y \sin x \, dy}\, dx$$

$$= \int_0^{\pi} \left(\frac{y^2}{2} \sin x \Big|_{y=1}^{y=2} \right) dx$$

$$= \int_0^{\pi} \left(2 \sin x - \frac{1}{2} \sin x \right) dx$$

$$= \int_0^{\pi} \frac{3}{2} \sin x \, dx = -\frac{3}{2} \cos x \Big|_0^{\pi}$$

$$= -\frac{3}{2} \cdot (-1) - \left(-\frac{3}{2} \cdot 1 \right)$$

$$= \frac{3}{2} + \frac{3}{2} = 3.$$

Exc #6 p 313

$$\int_1^9 \int_1^e \frac{\ln \sqrt{x}}{xy} \, dx \, dy$$

$$= \int_1^9 \int_1^e \underbrace{\frac{1}{2} \frac{\ln x}{xy}}_{\text{How?}} \, dx \, dy$$

First recall

$$\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2}+c = \frac{(\ln x)^2}{2}+c$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

go back to original $\iint$

$$\int_1^9 \int_1^e \frac{1}{2}\,\frac{\ln x}{x}\,\frac{1}{y}\,dx\,dy$$

$$= \int_1^9 \frac{1}{2y}\,\frac{(\ln x)^2}{2}\Big|_{x=1}^{x=e}\,dy = \int_1^9 \frac{1}{4y}\,(1^2 - 0^2)\,dy$$

$$= \int_1^9 \frac{1}{4y}\,dy = \frac{1}{4}\cdot \ln|y|\Big|_1^9 = \frac{1}{4}\,(\ln 9 - \cancel{\ln 1}^{\,0})$$

$$= \frac{\ln 9}{4}.$$

Exc # 8

(b) $\displaystyle\int_1^2 \int_0^3 (x+3y+1)\,dx\,dy$ by slicing $y=$ const.

(a) $\displaystyle\int_0^3 \int_1^2 (x+3y+1)\,dy\,dx$ by slicing $x=$ const.

Next page we'll calculate one:

$$\int_1^2 \int_0^3 (x+3y+1)\,dx\,dy$$

$$= \int_1^2 \left(\frac{x^2}{2} + 3xy + x \Big|_{x=0}^{x=3} \right) dy$$

$$= \int_1^2 \left(\left(\frac{9}{2} + 9y + 3 \right) - (0) \right) dy$$

$$= \int_1^2 \left(9y + \frac{15}{2} \right) dy$$

$$= \frac{9y^2}{2} + \frac{15}{2} y \Big|_1^2$$

$$= \left(\frac{9}{2} \cdot 4 + \frac{15}{2} \cdot 2 \right) - \left(\frac{9}{2} + \frac{15}{2} \right)$$

$$= (18 + 15) - 12 = 21$$