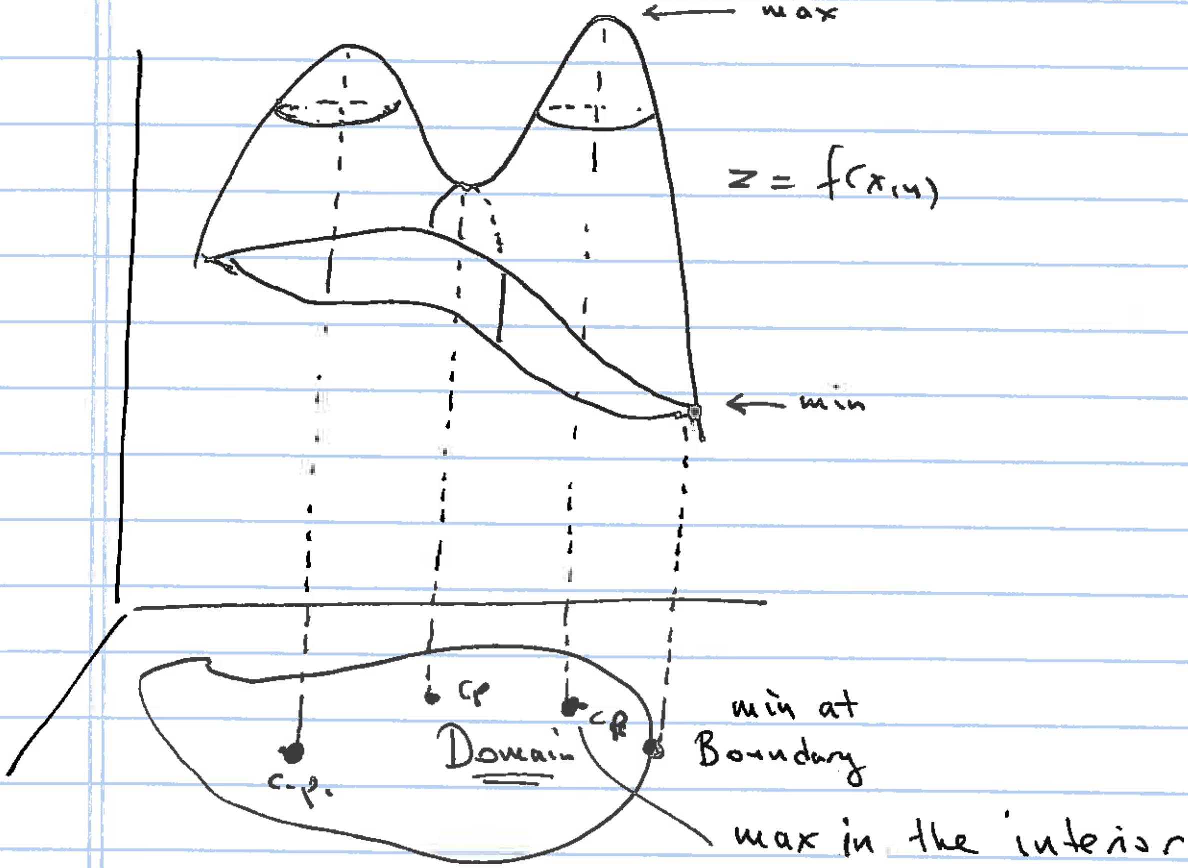
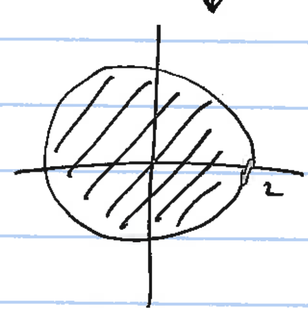
4.2 Continue



Ex 1 $f(x,y) = (x-1)^2 + y^2$

Find max and minimum values of f on the region $x^2+y^2 \le 4$.

- interior c.p. (First + 2^{nd} D.Test)
- non-diff'ble pts
- Boundary behavior



Interior

$$\nabla f = (2(x-1),\ 2y)$$

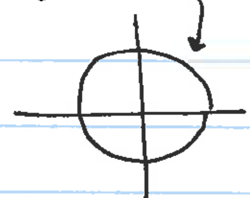
1^{st} D. Test $\quad \nabla f = 0 \iff x = 1$ and $y = 0$.

$(1,0)$ only interior c.p.

Boundary of $\{(x,y) \mid x^2+y^2 \le 4\}$ is

$\{(x,y) \mid x^2+y^2 = 4\}$

One can parametrize the boundary



$(x,y) = (2\cos t,\ 2\sin t) \qquad 0 \le t \le 2\pi$

Study f on the boundary:

$$f(x,y) = f(2\cos t,\ 2\sin t) = g(t)$$

$$= (2\cos t - 1)^2 + (2\sin t)^2$$

$$= 4\cos^2 t - 4\cos t + 1 + 4\sin^2 t$$

$$= 5 - 4\cos t$$

find its largest & smallest values

$g = 5 - 4\cos t \qquad 0 \le t \le 2\pi$

$g' = +4\sin t$

$g' = 0 \iff t = 0, \pi, 2\pi$

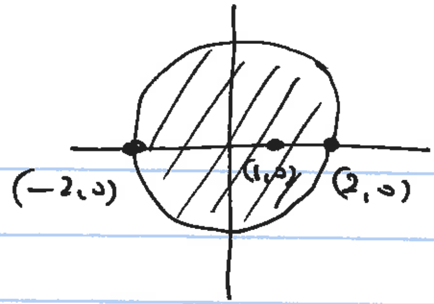
also endpts

$g(0) = (2,0)$

$g(\pi) = (-2,0)$

$g(2\pi) = (2,0)$

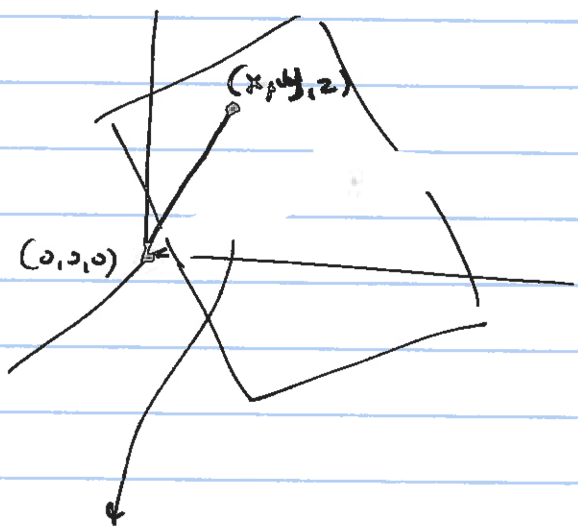
	List of c.p.	$f=(x-1)^2+y^2$	
Boundary	$(-2,0)$	9	max
interior	$(1,0)$	0	min.
Boundary	$(2,0)$	1	



Domain is compact.

$\Rightarrow \exists$ max / min values

Ex 2 Find the closest pt of the plane $x+3y-2z=7$ to the origin.



$$f(x,y,z)=\big(\text{distance of } (x,y,z) \text{ to } (0,0,0)\big)^2$$

$$f(x,y,z) = x^2+y^2+z^2 \qquad \text{subject to } (\underbrace{x+3y-2z=7}_{\text{Constraint.}})$$

$$x = 7-3y+2z$$

$$g(y,z)= (7-3y+2z)^2+y^2+z^2 \qquad \text{minimize}$$

$$\left.\begin{aligned} -\infty < y < \infty \\ -\infty < z < \infty \end{aligned}\right\} \text{ not compact.}$$

$g = g(y, z)$

$$g_y = 2y + 2(7-3y+2z)\cdot(-3) = 20y - 12z - 42$$

$$g_z = 2z + 2(7-3y+2z)\cdot 2 = -12y + 10z + 28$$

1st D.T.
Solve
$\nabla g = (0,0)$

$$\left\{\begin{array}{l} 20y - 12z = 42 \\ -12y + 10z = -28 \end{array}\right\}$$

$$\Rightarrow \cdots \Rightarrow \left\{\begin{array}{l} y = \frac{3}{2} \\ z = -1 \end{array}\right.$$

$x + 3y - 2z = 7 \Rightarrow x = \frac{1}{2}$.

[Extreme V. Thm
Does <u>not</u> apply.

$$Hg = \begin{bmatrix} 20 & -12 \\ -12 & 10 \end{bmatrix}$$

$$\left.\begin{array}{l} \text{Det} = 200 - 144 > 0 \\ g_{yy} = 20 > 0 \end{array}\right\}$$

local min.

g has a unique critical point, and it is a local min.
Since $\exists$ a unique closest pt of the plane to $\vec{0}$, the answer needs to be

$$\left(\frac{1}{2}, \frac{3}{2}, -1\right).$$