Oct 10, 2016

Continue 4.1

Ex #10 $\quad f = e^{2x+y} \qquad a = (0,0)$

$f(0,0) = 1$

$f_x = 2e^{2x+y} \qquad f_x(0,0) = 2$

$f_y = e^{2x+y} \qquad f_y(0,0) = 1$

$f_{xx} = 4e^{2x+y} \qquad f_{xx}(0,0) = 4$

$f_{xy} = f_{yx} = 2e^{2x+y} \qquad f_{xy}(0,0) = f_{yx}(0,0) = 2$

$f_{yy} = e^{2x+y} \qquad f_{yy}(0,0) = 1.$

$$p_1(x,y) = 1 + 2(x-0) + 1(y-0) = 1+2x+y$$

$$p_2(x,y) = 1+2x+y + \frac{1}{2}\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= 1+2x+\cancel{2}y + \frac{1}{2}\left[4x^2 + 2xy + 2yx + y^2\right]$$

$$= 1+2x+y + 2x^2 + 2xy + \frac{1}{2}y^2$$

To calculate: $P_3(e^{2x+y}, (0,0))$

	at $(0,0)$
$f_{xxx} = 8e^{2x+y}$	8
$f_{xxy} = 4e^{2x+y} = f_{xyx} = f_{yxx}$	4 (3 of them)
$f_{xyy} = 2e^{2x+y} = f_{yxy} = f_{yyx}$	2 (3 of them)
$f_{yyy} = e^{2x+y}$	1

$$P_3(e^{2x+y}, (0,0)) = 1 +$$

$$+ 2(x-0) + 1(y-0) +$$

$$+ \frac{1}{2}\left(4(x-0)^2 + 4(x-0)(y-0) + 1(y-0)^2 \right)$$

$$+ \frac{1}{6}\left(8(x-0)^3 + 12(x-0)^2(y-0) + 6(x-0)(y-0)^2 + 1(y-0)^3 \right)$$

Ex

$$f = x^3 + 6xyz + xy^2 - 7z$$

Want p_2, at $(1,2,0)$

	at $(1,2,0)$	
$f = x^3 + 6xyz + xy^2 - 7z$	5	
$f_x = 3x^2 + 6yz + y^2$	7	First order
$f_y = 6xz + 2xy$	4	
$f_z = 6xy - 7$	5	
$f_{xx} = 6x$	6	2nd order
$f_{xy} = 6z + 2y$	4	
$f_{yy} = 2x$	2	
$f_{xz} = 6y$	12	
$f_{yz} = 6x$	6	
$f_{zz} = 0$	0	

$$P_2 = 5 + \begin{bmatrix} 7 & 4 & 5 \end{bmatrix} \begin{bmatrix} x-1 \\ y-2 \\ z-0 \end{bmatrix} +$$

$$+ \frac{1}{2} \begin{bmatrix} x-1 & y-2 & z-0 \end{bmatrix} \begin{bmatrix} 6 & 4 & 12 \\ 4 & 2 & 6 \\ 12 & 6 & 0 \end{bmatrix} \begin{bmatrix} x-1 \\ y-2 \\ z-0 \end{bmatrix}$$

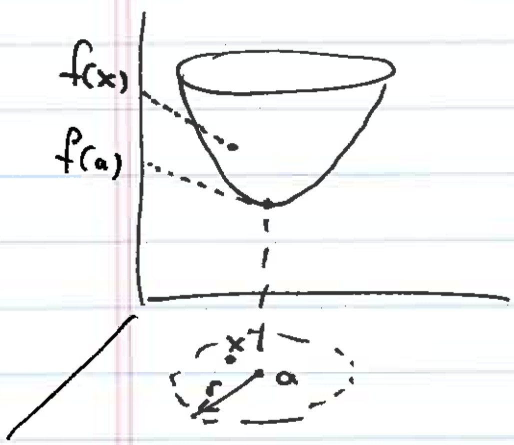
End of 4.1

4.2 Max/min.

Def Let $f : \mathbb{X} \subseteq \mathbb{R}^n \to \mathbb{R}^1$

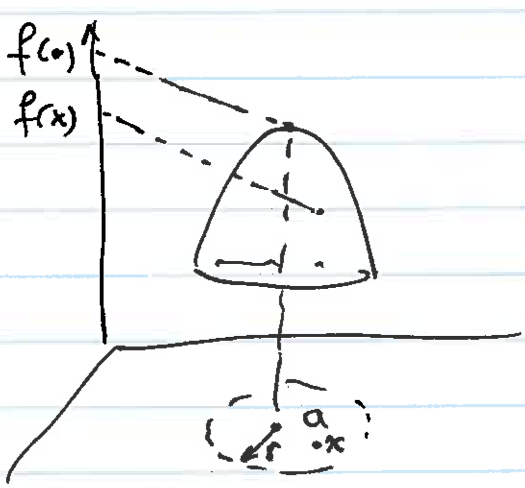
- f is said to have a local minimum at $a \in \mathbb{X}$ if $\exists r > 0$

$$\forall x \in \mathbb{X}, \text{ s.t. } |x-a| < r, \quad f(x) \geq f(a)$$



- f is said to have a local maximum at $a \in \mathbb{X}$ if $\exists r > 0$ s.t.

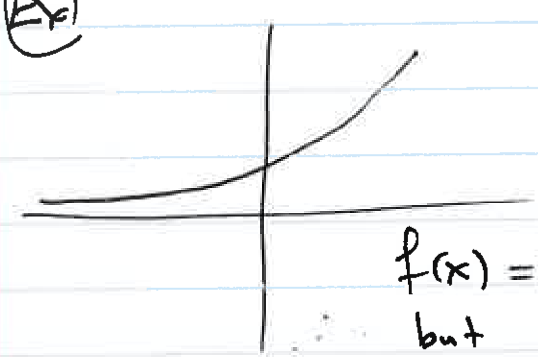
$$\forall x \in \mathbb{X}, \text{ s.t. } |x-a| < r, \quad f(x) \leq f(a)$$



f is said to have a local extremum at a if either a is a local max or a local min.

- f is said to have a global max at $a \in \bar{X}$ if For all $x \in \bar{X}$, $f(a) \geq f(x)$
- f is said to have a global min at a if For all $x \in \bar{X}$ $f(a) \leq f(x)$
- f is said to be bounded (from) above if $\exists M$ s.t. $\forall x \in \bar{X}$ $f(x) \leq M$
- f is said to be bounded (from) below if $\exists M$ s.t. $\forall x \in \bar{X}$ $f(x) \geq M$.

Ex

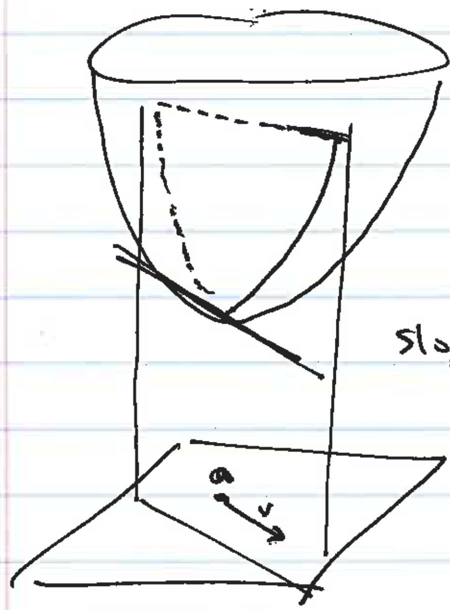


$f(x) = e^x \geq 0$ bounded below. but has no global/local min.

Thm: Let $f: \underline{X}^{\text{open}} \subseteq \mathbb{R}^n \to \mathbb{R}^1$,
Let $a \in \underline{X}$, Let a be a local extremum
Let f be diff'ble at a. Then

$$\nabla f(a) = \vec{0} \quad \left(Df(a) = [0 \cdots 0] \right)$$

Proof: Case 1 a is a local min.



$(D_v f)(a) =$

slope of the tangent of the section at a obtained by slicing the graph of $z = f(x,y)$ with a vertical plane thru a, parallel to v.

$$(D_v f)(a) = \lim_{h \to 0} \frac{f(a+hv) - f(a)}{h} = 0$$

since $f(a+hv)$ has a local min at $h=0$

In every direction v:

$$\forall_v \quad (D_v f)(a) = 0 = \nabla f(a) \cdot v$$

$$\Rightarrow \nabla f(a) = 0.$$

<u>Defn</u> Let $\mathbb{X} \subseteq \mathbb{R}^n \to \mathbb{R}^1$, $a \in \mathbb{X}$ is called a critical point if
either $\nabla f(a) = 0$, or
f is not diff'ble at a.