3.1

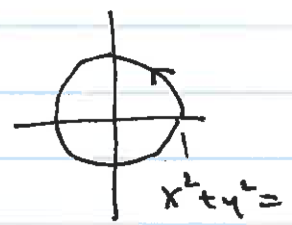
<u>Defn</u> A continuous function
$\vec{x} : \underset{\text{interval} \subseteq \mathbb{R}^1}{I} \longrightarrow \mathbb{R}^n$ is called a path

or a parametrized curve.

If $I = [a, b]$ then $\vec{x}(a)$, $\vec{x}(b)$ are the end points.

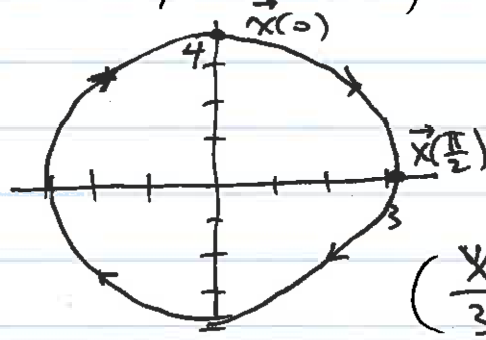
Ex . lines $\vec{x}(t) = (1,0,5) + t(1,6,-2)$
$x(t) = \vec{p}_0 + t\vec{v}_0$

- $\vec{x}(t) = (\underbrace{\cos t}_{x}, \underbrace{\sin t}_{y})$



- $\vec{x}(t) = (\overbrace{3\sin t}^{x}, \overbrace{4\cos t}^{y}) : \mathbb{R} \longrightarrow \mathbb{R}^2$

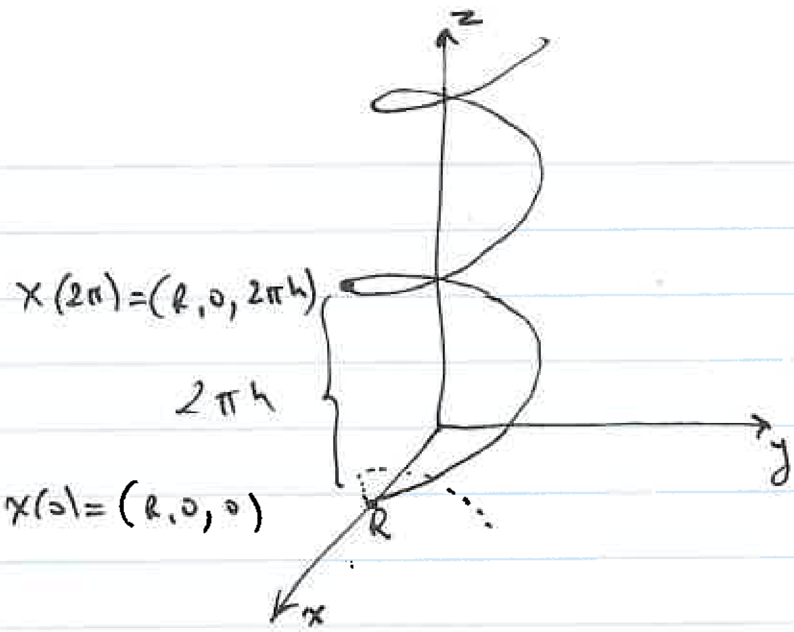
$\vec{x}(0) = (0,4)$
$\vec{x}(\frac{\pi}{2}) = (3,0)$



Ellipse

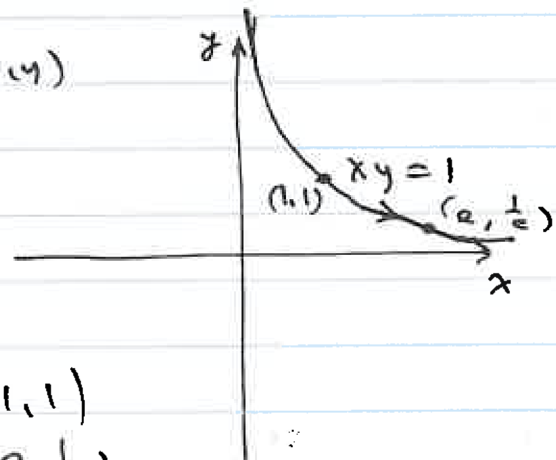
$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

- $\vec{x}(t) = (R\cos t, R\sin t, ht)$
Called <u>Helix</u>.



p200 Exc #2 $\vec{x}(t) = e^t \vec{i} + e^{-t} \vec{j} = (e^t, e^{-t})$

$\vec{x} = (x, y)$



$x = e^t > 0$

$y = e^{-t} > 0$

$xy = 1$

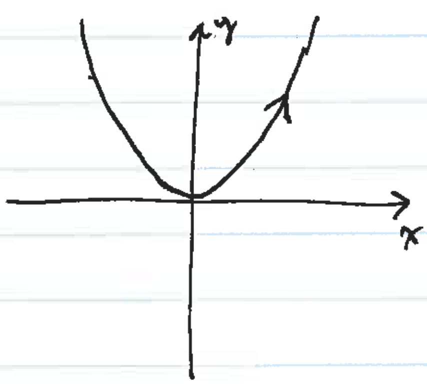
$\vec{x}(0) = (1, 1)$

$\vec{x}(1) = (e, \frac{1}{e})$

p200 #6 $\vec{x} = (t, t^2, t^3)$

Top view $(x, y) = (t, t^2)$

$(x, z) = (t, t^3)$

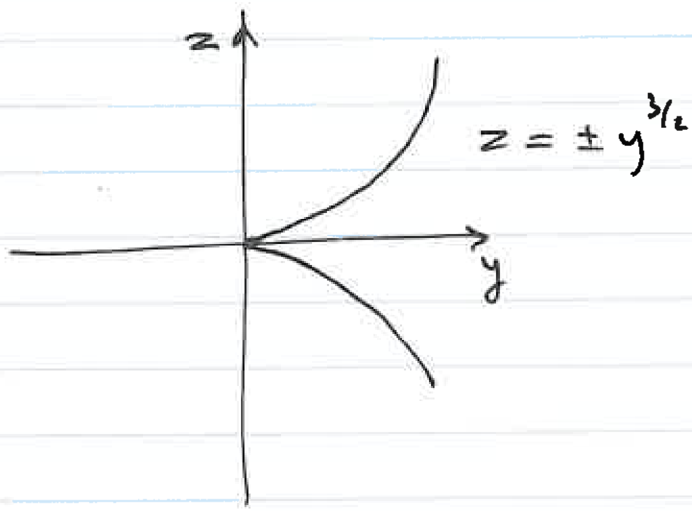


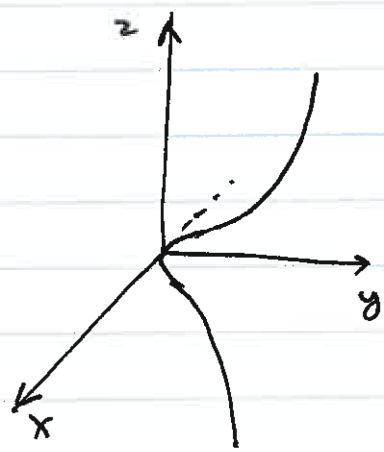
$$(y, z) = (t^2, t^3)$$
$$\quad\quad\quad\quad\quad \overset{\shortparallel}{y} \quad \overset{\shortparallel}{z}$$

$y = t^2$

$z = t^3$ $\quad$ $y^3 = z^2$

$z = \pm y^{3/2}$





<u>Defn</u> Let $\vec{x}: I \subseteq \mathbb{R}^1 \longrightarrow \mathbb{R}^n$ be a diff'ble parametrized curve

Velocity $v(t) = \dfrac{d\vec{x}}{dt} = x'(t) = \dot{x}(t)$

Speed $=$ $\|$velocity$\|$ $= \| x'(t) \|$

Acceleration $\dfrac{dv}{dt} = x''(t) = \ddot{x}(t)$

Unit tangent vector = direction of velocity

velocity = (unit tangent vector) $\cdot$ (speed)

$$x'(t) = \frac{x'(t)}{\|x'(t)\|} \qquad \|x'(t)\|$$

provided that $\|x'(t)\| \neq 0$.

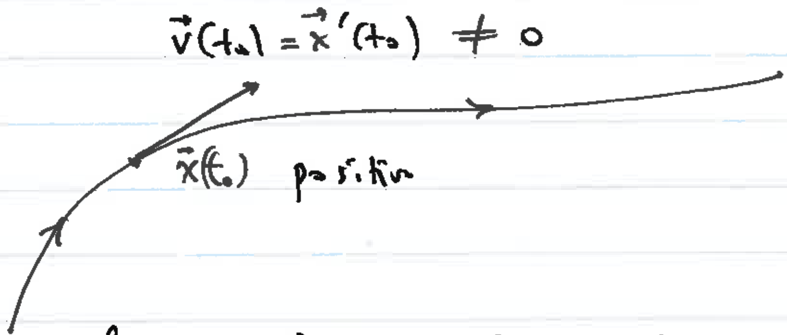
p200 <u>Exc</u> #8 $\vec{x}(t) = 5\cos t \; i + 3\sin t \; j$

velocity $x' = -5\sin t \; i + 3\cos t \; j$

speed $\|x'\| = \sqrt{25\sin^2 t + 9\cos^2 t}$

acceleration $x'' = -5\cos t \; i + (-3\sin t) \; j$

Tangent line to a parametric curve:



$$\underbrace{\ell(t) = \vec{x}(t_0) + \vec{x}\,'(t_0)\,(t - t_0)}_{\text{parametric line}}$$

#16 $4\cos t\,\vec{i} - 3\sin t\,\vec{j} + 5t\,\vec{k} = x(t)$

Tangent line at $t = \frac{\pi}{3}$.

$$x\left(\tfrac{\pi}{3}\right) = \left(4 \cdot \tfrac{1}{2},\ -3\tfrac{\sqrt{3}}{2},\ 5 \cdot \tfrac{\pi}{3}\right)$$

$$x'(t) = (-4\sin t,\ -3\cos t,\ 5)$$

$$x'\left(\tfrac{\pi}{3}\right) = \left(-\tfrac{4\sqrt{3}}{2},\ -3 \cdot \tfrac{1}{2},\ 5\right)$$

Tangent line

$$\ell(t) = \left(2, -\tfrac{3\sqrt{3}}{2}, \tfrac{5\pi}{3}\right) + \left(t - \tfrac{\pi}{3}\right)\left(-2\sqrt{3}, -\tfrac{3}{2}, 5\right)$$