Review ①
9/29/16

#4 Practice Test

$f(x,y,z) = (x^2 - y^2, xyz) \; : \; \mathbb{R}^3 \to \mathbb{R}^2$

$g(u,v) = (uv, u - 3v, u^2 + v^3) \; : \; \mathbb{R}^2 \to \mathbb{R}^3$

$$\underset{2\times 3}{Df} = \begin{bmatrix} 2x & -2y & 0 \\ yz & xz & xy \end{bmatrix}$$

$$\underset{3\times 2}{Dg} = \begin{bmatrix} v & u \\ 1 & -3 \\ 2u & 3v^2 \end{bmatrix}$$

$g(1,2) = (2, -5, 9)$

$$D(f \circ g)(1,2) = Df(g(1,2)) \cdot Dg(1,2)$$

$$= Df(2,-5,9) \cdot Dg(1,2)$$

$$= \underset{2\times 3}{\begin{bmatrix} 4 & 10 & 0 \\ -45 & 18 & -10 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 2 & 1 \\ 1 & -3 \\ 2 & 12 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} 18 & -26 \\ -92 & -219 \end{bmatrix}}$$

$$D(g \circ f)(1,2,3) = Dg(f(1,2,3)) \cdot Df(1,2,3)$$

$$f(x,y,z) = (x^2 - y^2,\ xyz)$$

$$f(1,2,3) = (-3,\ 6)$$

$$= \underset{3 \times 2}{\begin{bmatrix} 6 & -3 \\ 1 & -3 \\ -6 & 108 \end{bmatrix}} \underset{2 \times 3}{\begin{bmatrix} 2 & -4 & 0 \\ 6 & 3 & 2 \end{bmatrix}}$$

72
6
432

74
324

−12
648

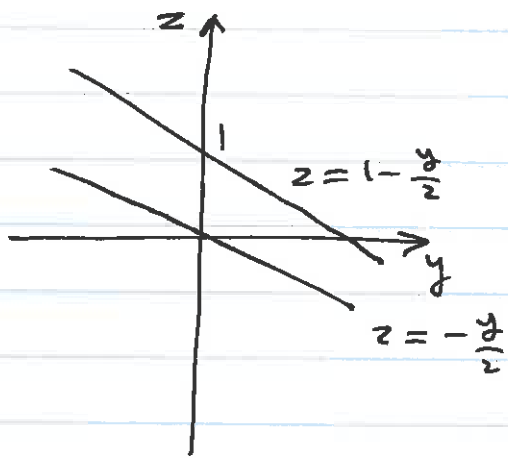
$$= \underset{3 \times 3}{\begin{bmatrix} -6 & -33 & -6 \\ -16 & -13 & -6 \\ 636 & 348 & 216 \end{bmatrix}}$$

Practice test

#5 $z = f(x,y) = x^2 - \frac{y}{2}$

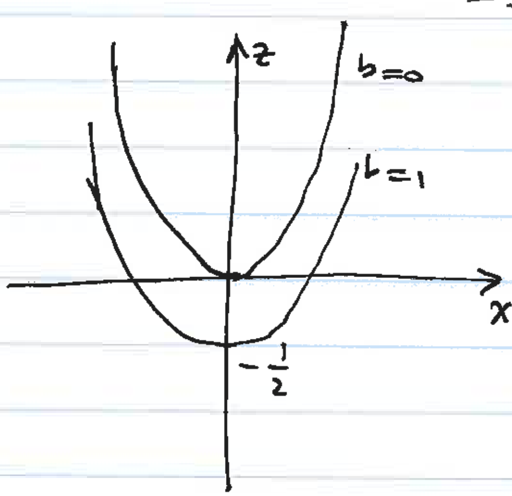
(a) $f(a,y)$ $\quad z = f(0,y) = -\frac{y}{2}$

$z = f(1,y) = 1 - \frac{y}{2}$



(b) $f(x,b)$ $\quad z = f(x,0) = x^2$

$z = f(x,1) = x^2 - \frac{1}{2}$

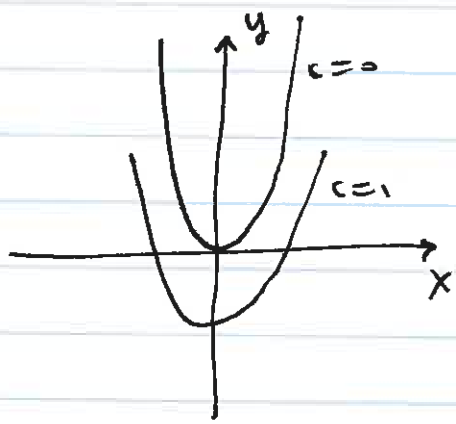


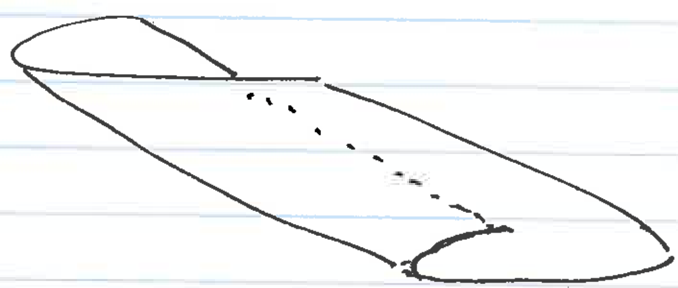
(c) $f(x,y) = c$ $\quad f(x,y) = 0 = x^2 - \frac{y}{2}$

$f(x,y) = 1 = x^2 - \frac{y}{2}$

$$x^2 - \frac{y}{2} = 0 \implies x^2 = \frac{y}{2}$$
$$\implies y = 2x^2.$$

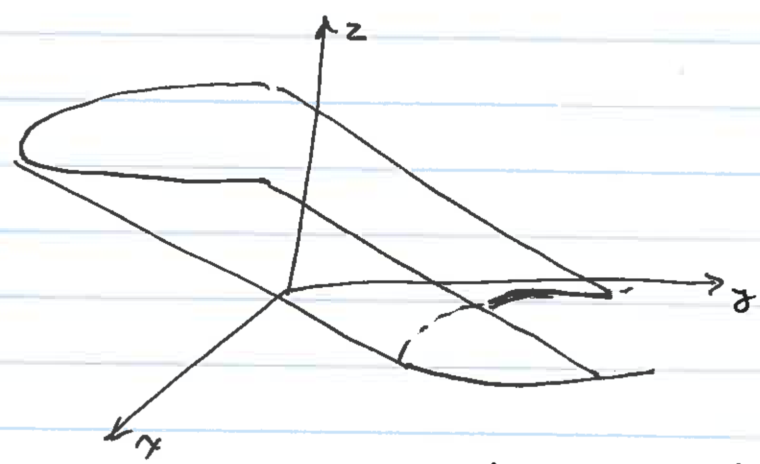
$$x^2 - \frac{y}{2} = 1 \implies x^2 - 1 = \frac{y}{2}$$
$$y = 2x^2 - 2$$





$z = f(x,y)$

$z = x^2 - \frac{y}{2}$



parabolic cylinder

#8 $g(x,y) = xe^{-y}$

(a) $\nabla g = (e^{-y}, -xe^{-y}) = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right)$

$Dg = \begin{bmatrix} e^{-y} & -xe^{-y} \end{bmatrix}$

$h(x,y) = g(a,b) + \frac{\partial g}{\partial x}(a,b)(x-a) + \frac{\partial g}{\partial y}(a,b)(y-b)$

$(a,b) = (3,0)$

$g(3,0) = 3$

$\frac{\partial g}{\partial x}(3,0) = 1$

$\frac{\partial g}{\partial y}(3,0) = -3$

(b) $z = h(x,y) = 3 + 1 \cdot (x-3) + (-3)(y-0)$

(c) $g(3.1, 0.3)$

$z = 3 + 1 \cdot (3.1 - 3) + (-3)(0.3 - 0)$

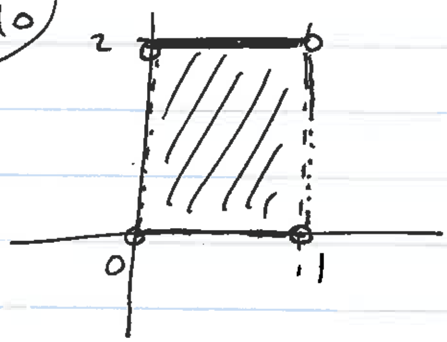
$= 3 + 0.1 - 0.9$

$= 2.2$

(d) $\nabla g(3,0) = (1, -3)$ $\qquad \frac{i - 3j}{\sqrt{10}}$

Practice test

#10



$0 < x < 1$

$0 \le y \le 2$

$\text{Bdry} = \{(x,y) \mid (0 \le x \le 1 \ \& \ y=0)$

or

$(0 \le x \le 1 \ \& \ y = 2)$

or

$(0 \le y \le 2 \ \& \ x = 0)$

or

$(0 \le y \le 2 \ \& \ x = 1)\}$

NOT open

NOT closed

(i) $\lim_{t \to 2} \left(t^2 + 2t, \frac{t^2-1}{t-1}, e^t\right) = (8, 3, e^2)$

(ii) ⊛ $\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ Along x-axis $(x,0)$

$\lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1$

Along y-axis $(0,y)$ $\lim_{(0,y)\to(0,0)} \frac{0}{y^2} = 0$ $\neq$.

Since approaching $(0,0)$ along y-axis and along x-axis gives different limits, ⊛ DNE.

iii)

$$\lim_{(x,y)\to(1,1)} \frac{xy-x}{1-y^2} = \lim_{(x,y)\to(1,1)} \frac{x\,\overset{-1}{\cancel{(y-1)}}}{\cancel{(1-y)}(1+y)}$$

$$= \lim_{(x,y)\to(1,1)} \frac{-x}{1+y} = -\frac{1}{2}$$

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$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy-xz+yz}{x^2+y^2+z^2} \qquad \text{DNE}$$

$$\left.\begin{array}{l} \displaystyle\lim_{(x,0,0)\to(0,0,0)} \frac{0}{x^2} = 0 \\[2ex] \displaystyle\lim_{(x,x,x)\to(0,0,0)} \frac{x^2-x^2+x^2}{x^2+x^2+x^2} = \frac{1}{3} \end{array}\right) \neq$$