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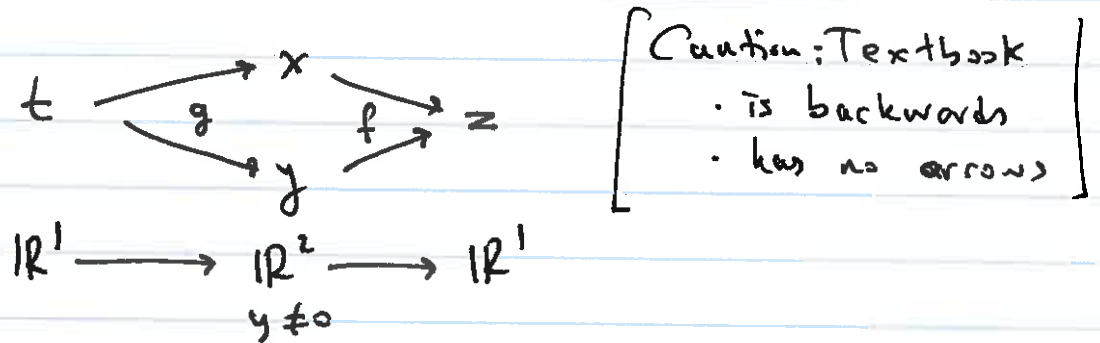
2.5 Chain Rule Open form.

①

Ex 1

$$g(t) = (t^2, t^3) = (x, y)$$

$$z = f(x, y) = \cos x^2 y + \frac{x}{y}$$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial t}$$

$$= \left(2xy(-\sin x^2 y) + \frac{1}{y} \right) \cdot 2t + \left(x^2(-\sin x^2 y) - \frac{x}{y^2} \right) \cdot 3t^2$$

$$\cdot \left. \frac{dz}{dt} \right|_{t=2} = \left(64(-\sin 144) + \frac{1}{8} \right) \cdot 4 + \left(16(-\sin 144) - \frac{1}{16} \right) \cdot 12$$

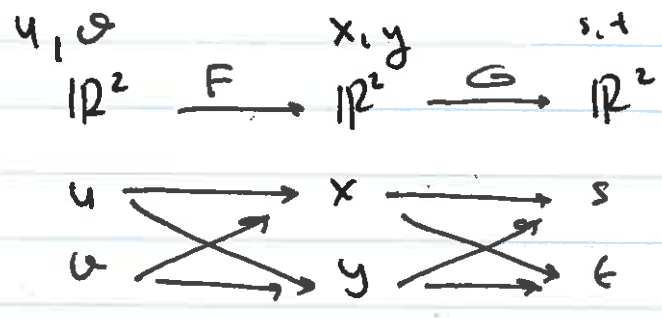
$t=2 \Rightarrow x=4$ since $x=t^2$
 $y=8$ $y=t^3$

Write As a function of t only $x=t^2$ $y=t^3$

$$\cdot \frac{dz}{dt} = \left(2t^5(-\sin t^7) + \frac{1}{t^3} \right) \cdot 2t + \left(t^4(-\sin t^7) - \frac{t^2}{t^6} \right) 3t^2$$

Ex2

$$\begin{aligned}
 x &= u + 3v^2 & s &= x^2 - 3xy \\
 y &= u^2 - 4v & t &= x^3 + e^y
 \end{aligned}$$



$$\frac{\partial s}{\partial u} = \frac{\partial s}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial s}{\partial y} \frac{\partial y}{\partial u}$$

$$\frac{\partial s}{\partial v} = \frac{\partial s}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial s}{\partial y} \frac{\partial y}{\partial v}$$

$$\frac{\partial t}{\partial u} = \frac{\partial t}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial t}{\partial y} \frac{\partial y}{\partial u}$$

$$\frac{\partial t}{\partial v} = \frac{\partial t}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial t}{\partial y} \frac{\partial y}{\partial v}$$

Compare

$$\begin{aligned}
 F(u, v) &= (x, y) \\
 G(x, y) &= (s, t)
 \end{aligned}$$

$$(s, t) = (G \circ F)(u, v)$$

$$D(G \circ F) = DG \cdot DF$$

$$\begin{bmatrix} \frac{\partial s}{\partial u} & \frac{\partial s}{\partial v} \\ \frac{\partial t}{\partial u} & \frac{\partial t}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{\partial s}{\partial x} & \frac{\partial s}{\partial y} \\ \frac{\partial t}{\partial x} & \frac{\partial t}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

(3)

$$\begin{aligned} \underbrace{F} \\ x &= u + 3v^2 \\ y &= u^2 - 4v \end{aligned}$$

$$\underbrace{G} \\ s &= x^2 - 3xy \\ t &= x^3 + e^y$$

$$\frac{\partial x}{\partial u} = 1 + 0$$

$$\frac{\partial s}{\partial x} = 2x - 3y$$

$$\frac{\partial x}{\partial v} = 6v$$

$$\frac{\partial s}{\partial y} = -3x$$

$$\frac{\partial y}{\partial u} = 2u$$

$$\frac{\partial t}{\partial x} = 3x^2$$

$$\frac{\partial y}{\partial v} = -4$$

$$\frac{\partial t}{\partial y} = e^y$$

$$DF = \begin{bmatrix} 1 & 6v \\ 2u & -4 \end{bmatrix} \quad DG = \begin{bmatrix} 2x-3y & -3x \\ 3x^2 & e^y \end{bmatrix} = DG$$

$$F(u, v) = (x, y)$$

$$G(x, y) = (s, t)$$

$$D(G \circ F) = DG \cdot DF$$

$$\begin{bmatrix} 2x-3y & -3x \\ 3x^2 & e^y \end{bmatrix} \begin{bmatrix} 1 & 6v \\ 2u & -4 \end{bmatrix}$$

$$= \begin{bmatrix} (2x-3y) + (-3x)(2u) & (2x-3y)6v + 12x \\ 3x^2 + 2u e^y & 18x^2v - 4e^y \end{bmatrix}$$

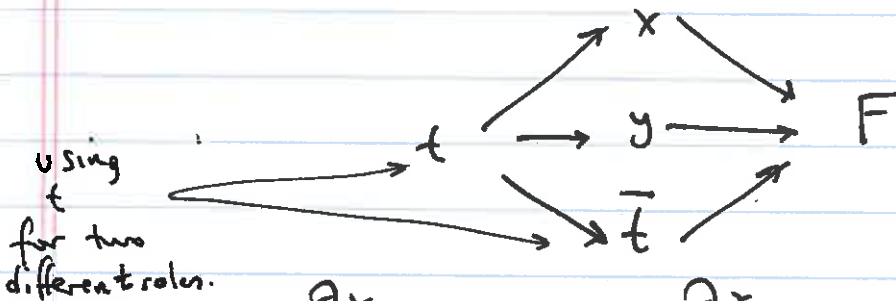
Ill-posed questions.

Ex $F(x, y, t) = x^2 t + xy t^2.$

$$x = t^2$$

$$y = t^3$$

$$\frac{d}{dt} F(x, y, t) = ?$$



Use t, \bar{t} to separate the roles.

$$\frac{\partial x}{\partial \bar{t}} = 0$$

$$\frac{\partial x}{\partial t} = 2t$$

$$\frac{d}{dt} F(x, y, t) = \frac{\partial F}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial F}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial F}{\partial \bar{t}} \cdot \frac{\partial \bar{t}}{\partial t}$$

$$= (2x\bar{t} + y\bar{t}^2) \cdot 2t +$$

$$+ (x\bar{t}^2) \cdot 3t^2 +$$

$$+ (x^2 + 2xy\bar{t}) \cdot 1$$

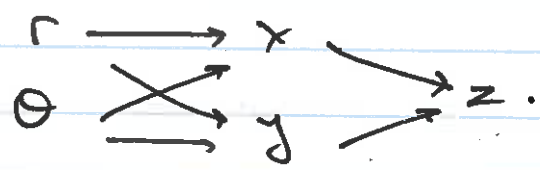
Ex # 30 p156

$$z = f(x, y)$$

Show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2$

where $\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$

Soln:



$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial r} = \frac{\partial z}{\partial x} \cdot \cos \theta + \frac{\partial z}{\partial y} \cdot \sin \theta$$
$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial \theta} = \frac{\partial z}{\partial x} (-r \sin \theta) + \frac{\partial z}{\partial y} r \cos \theta$$

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x} \cos \theta + \frac{\partial z}{\partial y} \sin \theta\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial x} (-r \sin \theta) + \frac{\partial z}{\partial y} r \cos \theta\right)^2$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 \cos^2 \theta + 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \cos \theta \sin \theta + \left(\frac{\partial z}{\partial y}\right)^2 \sin^2 \theta +$$

$$+ \frac{1}{r^2} \left(\left(\frac{\partial z}{\partial x}\right)^2 r^2 \sin^2 \theta - 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \sin \theta \cos \theta r^2 + \left(\frac{\partial z}{\partial y}\right)^2 r^2 \cos^2 \theta \right)$$

Factor & cancel r^2's.

$$= \left(\frac{\partial z}{\partial x}\right)^2 \cos^2 \theta + 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \cos \theta \sin \theta + \left(\frac{\partial z}{\partial y}\right)^2 \sin^2 \theta +$$

$$+ \left(\frac{\partial z}{\partial x}\right)^2 \sin^2 \theta - 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \cos \theta \sin \theta + \left(\frac{\partial z}{\partial y}\right)^2 \cos^2 \theta$$

$$= \left(\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right) \underbrace{(\cos^2 \theta + \sin^2 \theta)}_1$$

$$= \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 \quad \#$$