Exc # 2

$$f(x,y) = e^{x^2+y^2}$$

$$\frac{\partial f}{\partial x} = 2x \cdot e^{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = 2y \cdot e^{x^2+y^2}$$

Caution:

$$F(x,y,z) = e^{x^2+y^2}$$

$$\frac{\partial F}{\partial z} = 0 \qquad \frac{\partial F}{\partial x} = 2x\, e^{x^2+y^2} \qquad \frac{\partial F}{\partial y} = 2y\, e^{x^2+y^2}$$

Exc # 4 $f(x,y) = \dfrac{x^3 - y^2}{1+x^2+3y^4}$

$$\frac{\partial f}{\partial x} = f_x = \frac{(3x^2)(1+x^2+3y^4) - (2x)(x^3-y^2)}{(1+x^2+3y^4)^2}$$

$$\frac{\partial f}{\partial y} = f_y = \frac{(-2y)(1+x^2+3y^4) - (12y^3)(x^3-y^2)}{(1+x^2+3y^4)^2}$$

Example $f = (x^2+y^3)(x^4-y^5)$

$$\frac{\partial f}{\partial x} = f_x = 2x(x^4-y^5) + (4x^3)(x^2+y^3).$$

$$\frac{\partial f}{\partial y} = f_y = (3y^2)(x^4-y^5) + (-5y^4)(x^2+y^3)$$

Exc #6 $\quad f(x,y) = \ln(x^2+y^2)$

$$f_x = \frac{2x}{x^2+y^2}$$

$$f_y = \frac{2y}{x^2+y^2}$$

Example $f(x,y) = x^2y\cos(x+3y+y^2)$

$$\frac{\partial f}{\partial x} = f_x = 2xy\cos(x+3y+y^2) + x^2y\left(-\sin(x+3y+y^2)\right)\cdot 1$$

$$f_y = x^2\cos(x+3y+y^2) + x^2y\left(-\sin(x+3y+y^2)\right)\cdot(3+2y)$$

Example $\quad F(x,y,z) = 2x^2y - 7xz^2 + ye^z$

$$\frac{\partial F}{\partial x} = 4xy - 7z^2$$

$$\frac{\partial F}{\partial y} = 2x^2 + e^z$$

$$\frac{\partial F}{\partial z} = -14xz + ye^z$$

Ex

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

- $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE
- $f(x,y)$ is not continuous at $(0,0)$

$$\frac{\partial f}{\partial x} = \frac{y(x^2+y^2) - 2x \cdot xy}{(x^2+y^2)^2} \quad \text{when } (x,y) \neq (0,0)$$

$$= \frac{yx^2 + y^3 - 2x^2y}{(x^2+y^2)^2} = \frac{y^3 - x^2y}{(x^2+y^2)^2} = \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(0,0) = \frac{d}{dx} f(x,0)\Big|_{x=0} = \frac{d}{dx} \frac{0}{x^2+0}\Big|_{x=0} = \frac{d}{dx} \frac{0}{x^2}\Big|_{x=0}$$

$$= \left(\frac{d}{dx} 0\right)\Big|_{x=0} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \frac{d}{dy} f(0,y)\Big|_{y=0} = \frac{d}{dy} 0 = 0$$

In Summary:

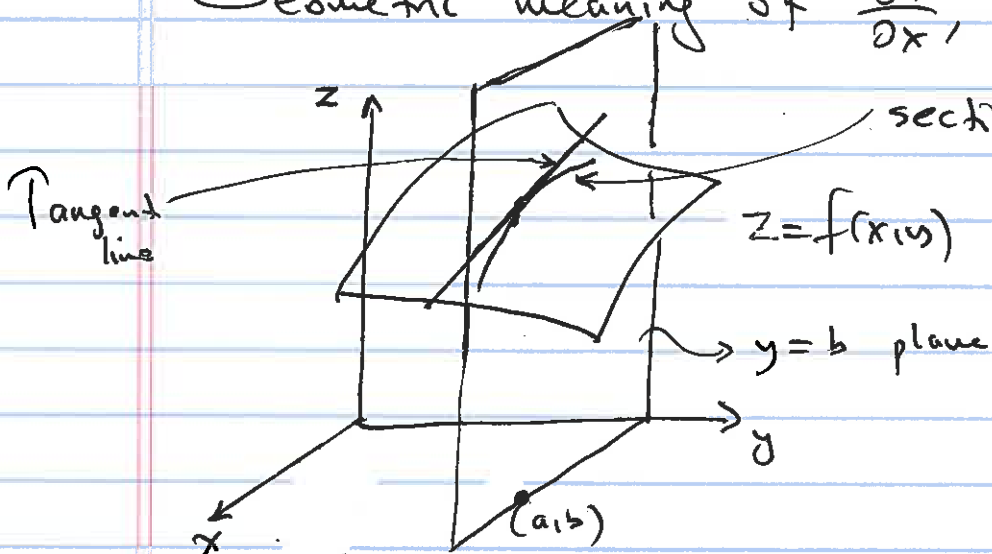
$$\frac{\partial f}{\partial x}(0,0) = 0 \neq \lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y) \leftarrow \text{DNE}$$

$$\frac{\partial f}{\partial y}(0,0) = 0 \neq \lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial y}(x,y) \leftarrow \text{DNE}$$

$$\lim_{(x,y)\to(0,0)} f(x,y) \quad \text{DNE}$$

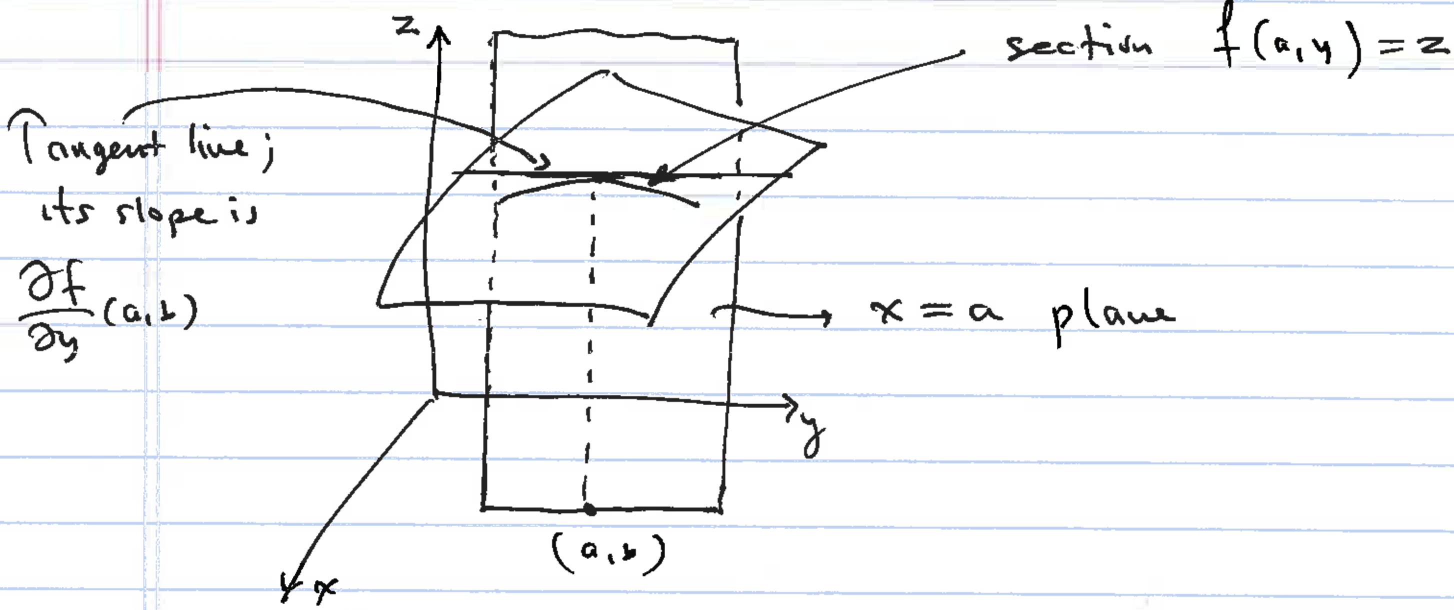
f is not continuous at $(0,0)$

Geometric meaning of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ at (a,b)



$$\frac{\partial f}{\partial x}(a,b) = \frac{d}{dx} f(x,b)\Big|_{x=a} = \text{slope of the section of the graph of } z=f(x,y) \text{ by the plane } y=b, \text{ at } x=a.$$

$$\frac{\partial f}{\partial y}(a,b) = \frac{d}{dy} f(a,y)\Big|_{y=b}$$



Defn Let $f: \mathbb{X}^{open} \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$, $(a,b) \in \mathbb{X}$.
f is called diff'ble at (a,b) if

1) $\frac{\partial f}{\partial x}(a,b)$, $\frac{\partial f}{\partial y}(a,b)$ both exist and finite,

and

2) $$h(x,y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b)$$

is a linear approximation of f such that

$$\lim_{(x,y)\to(a,b)} \frac{|f(x,y) - h(x,y)|}{\|(x,y)-(a,b)\|} = 0$$

Defn $z = h(x,y)$ is called the tangent plane $z = f(x,y)$ at $(a,b,f(a,b))$