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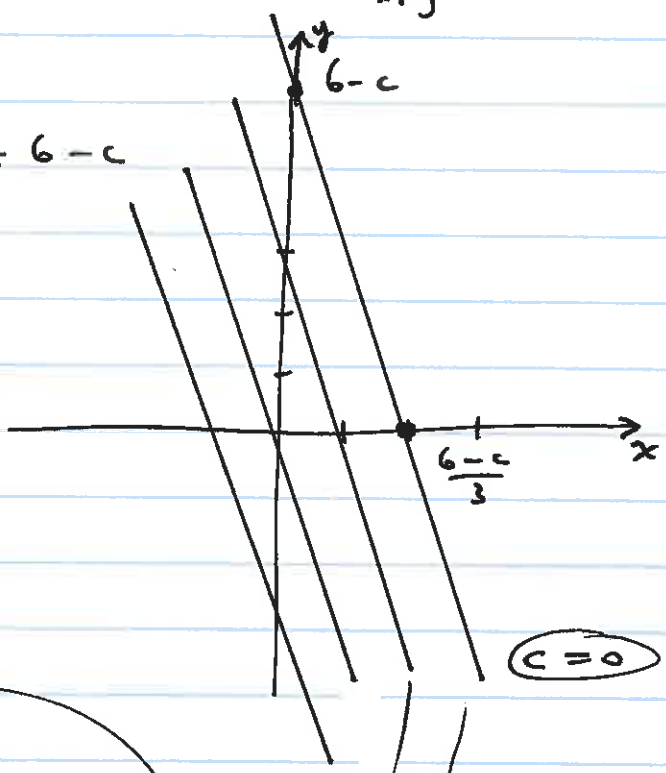
Example 6

$h(x,y) = 6 - 3x - y : \mathbb{R}^2_{x,y} \longrightarrow \mathbb{R}_z$

$6 - 3x - y = c$

$y = -3x + 6 - c$

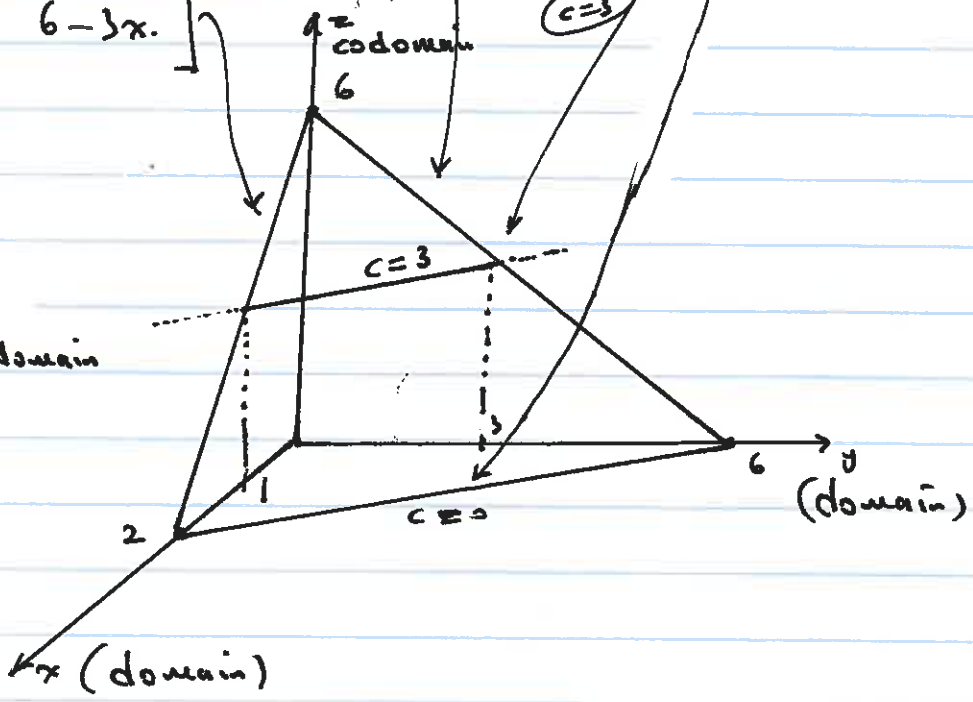


Level sets
Implicit graphs
in the Domain

Section

$x = 0$ (yz plane)
$z = h(0,y) = 6 - y$

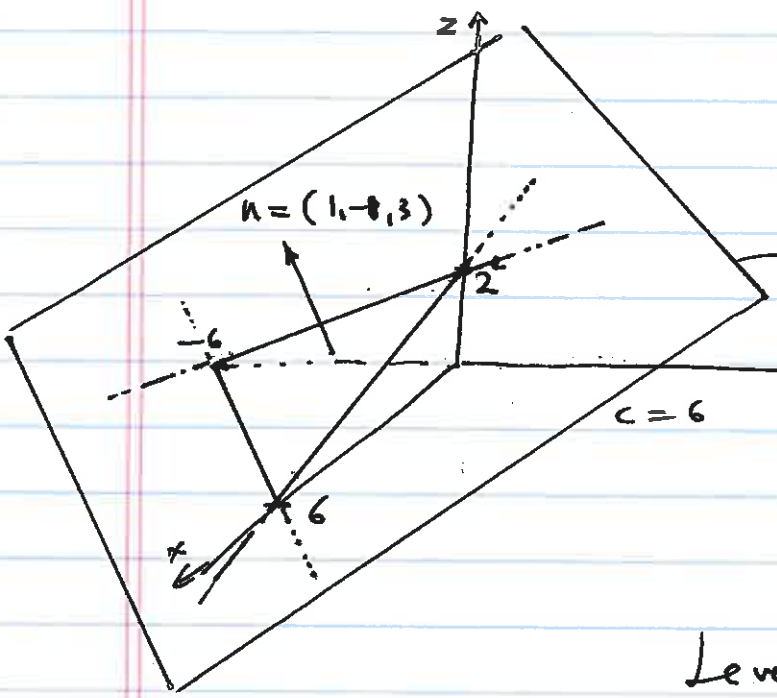
$y = 0$ (xz plane)
$z = h(x,0) = 6 - 3x.$



(Explicit) Graph
$\subseteq$ Domain x Codomain

Examples for $f: \mathbb{R}^3_{x,y,z} \to \mathbb{R}$ graph of $f \subseteq \mathbb{R}^4$
level sets $f \subseteq \mathbb{R}^3$.

Exc #32 $f(x,y,z) = x - y + 3z$.

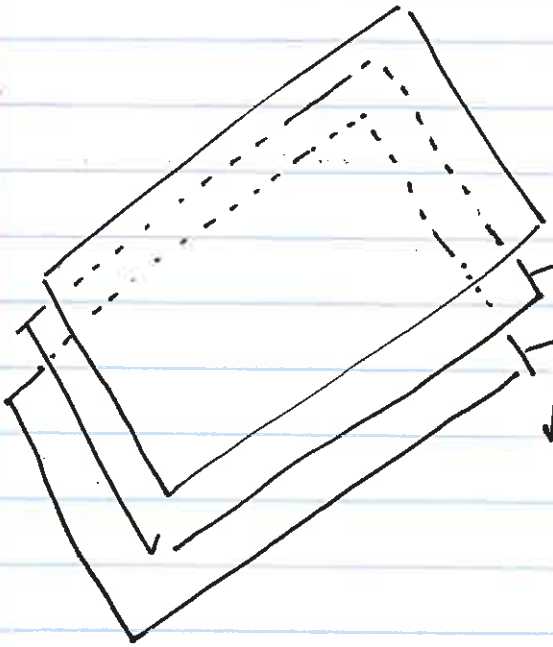


$c = 6 = f(x,y,z) = x - y + 3z$

f takes all plane to 6.

Implicit graphs $\subseteq$ Domain $\mathbb{R}^3_{x,y,z}$.

Level sets are:
$c = f(x,y,z) = x - y + 3z$
family of parallel planes
with common normal $(1,-1,3)$



↑ higher c

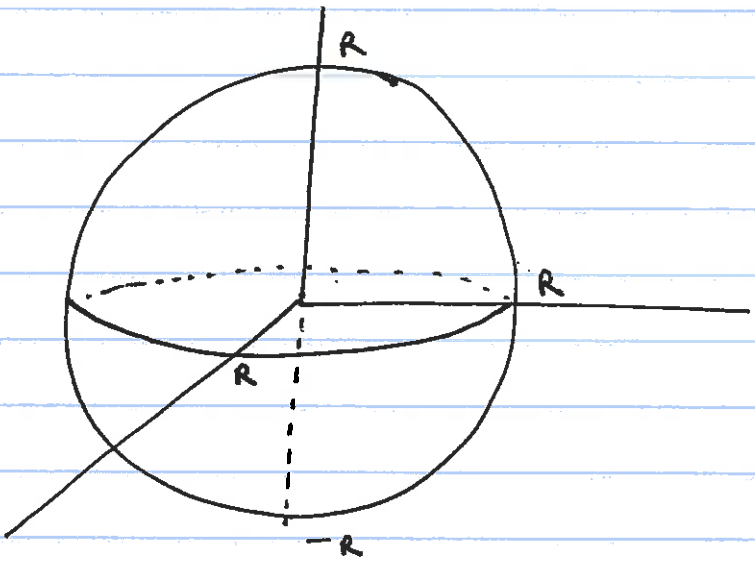
f takes all plane to value c_1

f takes all plane to value c_2

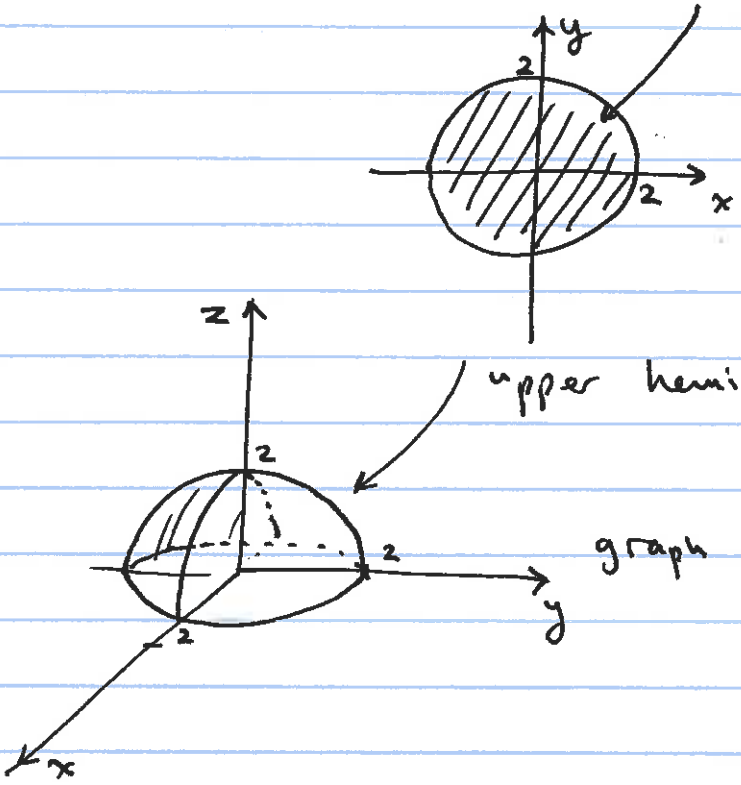
↓ lower c

Spheres:

$$x^2 + y^2 + z^2 = R^2$$



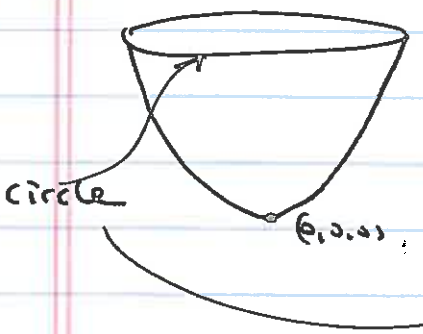
$$f(x,y) = \sqrt{4 - x^2 - y^2} \; : \; \underbrace{\{(x,y) \mid x^2 + y^2 \leq 4\}}_{\text{Domain of } f} \longrightarrow \mathbb{R}$$



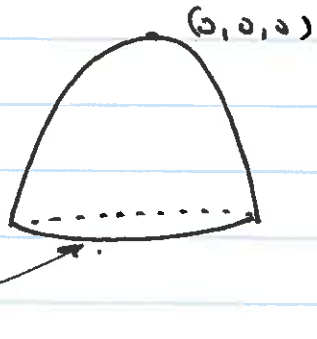
Note: Projection of graph to xy plane gives domain.

Basic surfaces to know:

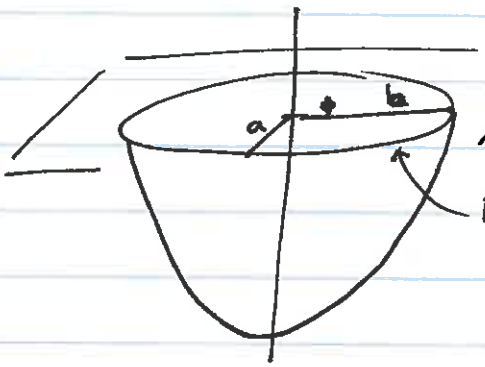
⊙ $z = x^2 + y^2$



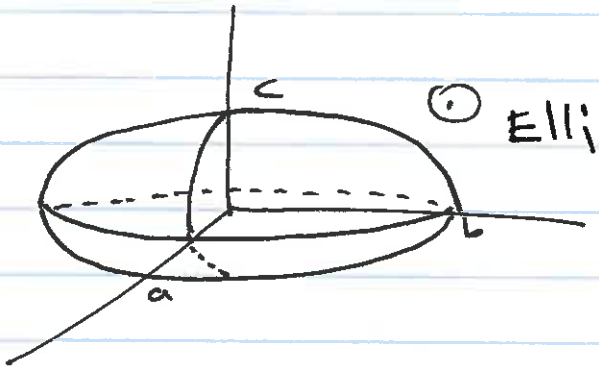
⊙ $z = -(x^2+y^2)$



⊙ $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$



⊙ $x^2 + y^2 + z^2 = R^2$ sphere



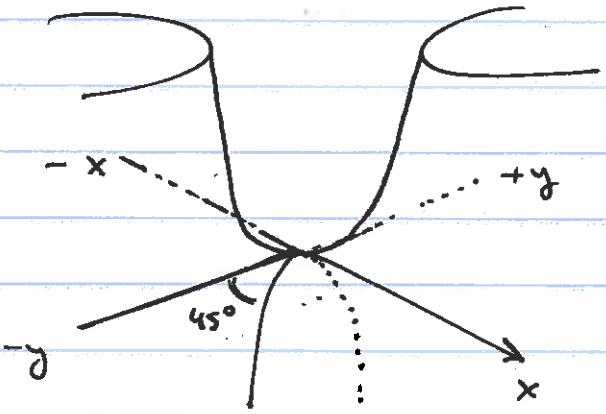
⊙ Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

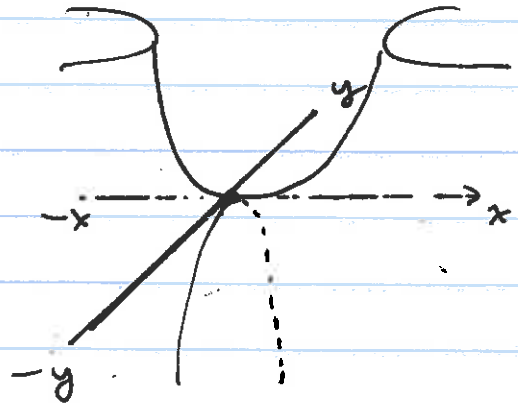
⊙ $z = 2xy$ and $z = x^2 - y^2$

both Saddles

one obtains the other by 45° rotation about z-axis:

Be careful where the x y axes are.





$z = 2xy$

The graph contains both x and y axes.

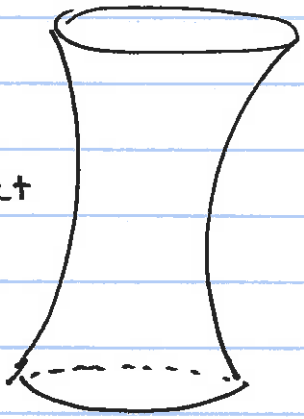
$z = x^2 - y^2$

x axes under graph

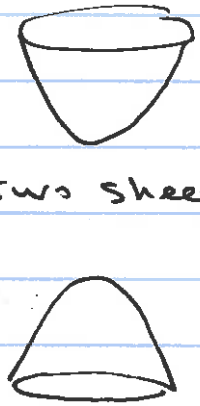
y axis over graph except (0,0,0)

Hyperboloids

one sheet



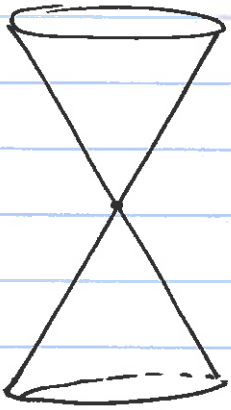
two sheets



$x^2 + y^2 - z^2 = 1$

$x^2 + y^2 - z^2 = -1$

Cone:



$x^2 + y^2 - z^2 = 0$

See pages 94-95 of the textbook.