Example 2

$$g(\underbrace{x, y, z}_{3 \,=\, \#\text{ variables}}) = (x+y,\ x^2+y^3-z)$$

each called a component function

\# components = 2

$$g: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

Is g 1-1? No $\left.\begin{array}{l} g(-1,1,2) = (0,0) \\ g(1,-1,0) = (0,0) \end{array}\right\}$ $(-1,1,2) \neq (1,-1,0)$

Is g onto $\mathbb{R}^2$? YES $\forall (a,b) \in \mathbb{R}^2$

$$g(0, a, a^3-b) = (a,b)$$

End of 9/7/16

9/8/16

GRAPHS : $f: X \subseteq \mathbb{R}^n \longrightarrow Y \subseteq \mathbb{R}^m$

(Explicit) Graph $f = \{ (\vec{x}, \vec{f(x)}) \in \mathbb{R}^n \times \mathbb{R}^m \mid \vec{x} \in X \}$
$\subseteq \mathbb{R}^n \times \mathbb{R}^m$

$n = \#$ variables
$m = \#$ components.

Parametric graph $f = \{ f(\vec{x}) \in \mathbb{R}^m \mid x \in X \subseteq \mathbb{R}^n \}$
$\subseteq \mathbb{R}^m$

Implicit graph : $\forall b \in Y$
$\{ x \in X \subseteq \mathbb{R}^n \mid f(x) = b \} \subseteq \mathbb{R}^n$

Also known as

Contour graphs / Inverse Images
Level sets /

Example 3 $f(x,y) = x^2 + y^2 - 1 \; : \; \mathbb{R}^2 \to \mathbb{R}^1$

Given a number $c \in \mathbb{R} = $ codomain

what is

$$f^{-1}(c) = L_c = \{ (x,y) \mid f(x,y) = c \}$$

level set for value c / contour curve

$$L_c = \{ (x,y) \mid x^2 + y^2 - 1 = c \}$$

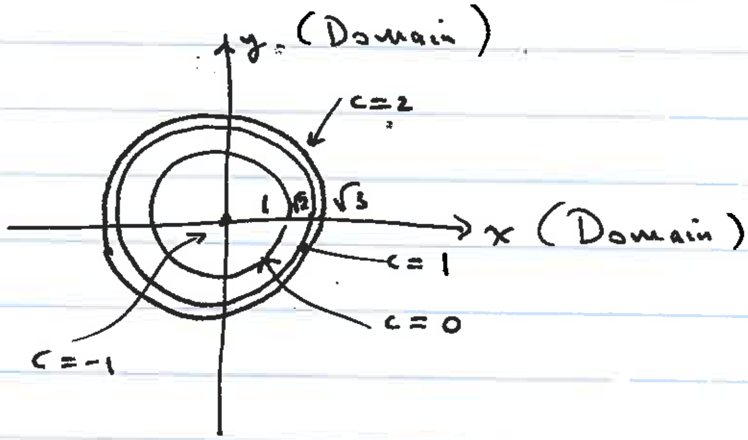
$$= \{ (x,y) \mid x^2 + y^2 = 1 + c \}$$

$\emptyset$ if $c < -1$.

$\{(0,0)\}$ if $c = -1$

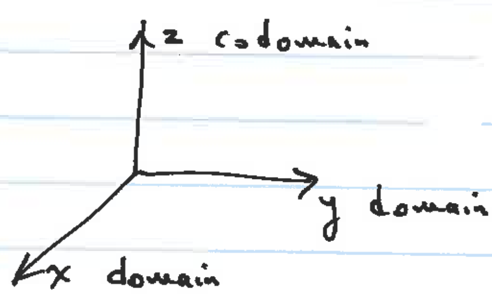
circle of radius $\sqrt{1+c}$ if $c > -1$.

Level sets:

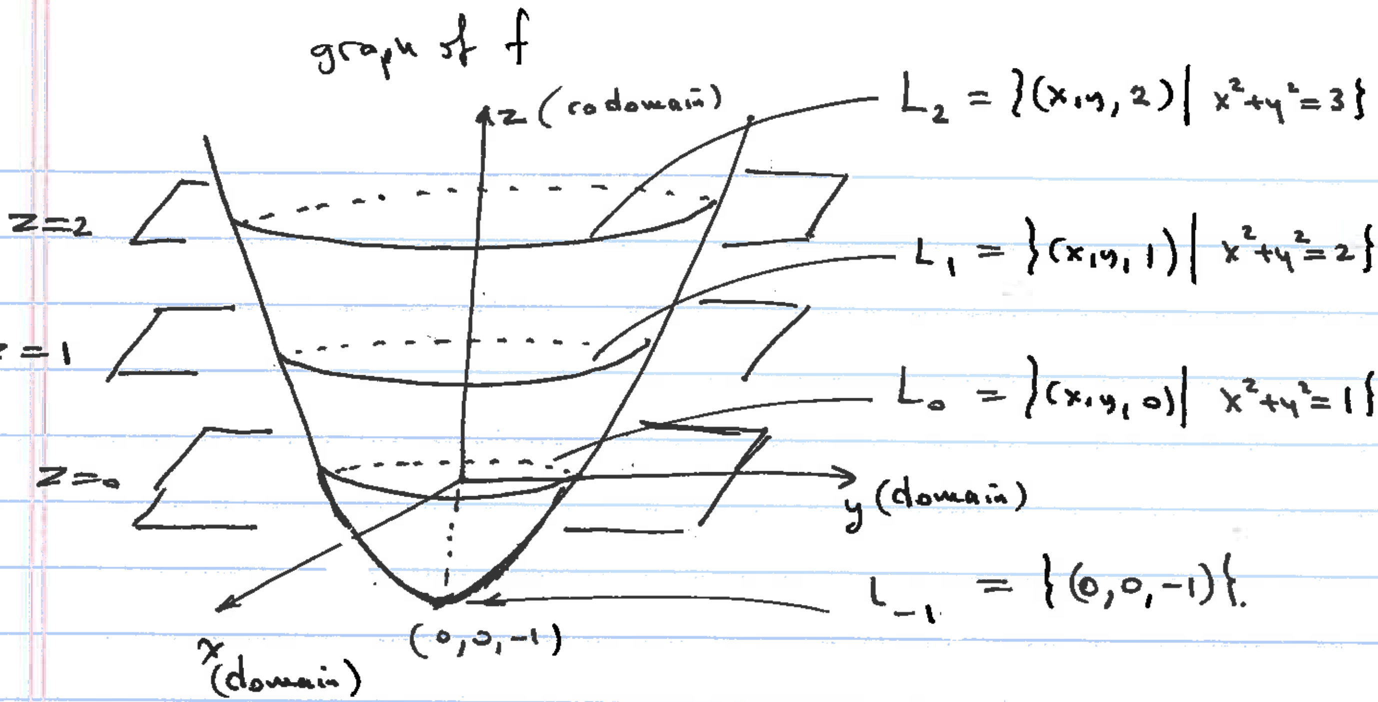


What is the graph of f.?

① it lies in $\mathbb{R}^2 \times \mathbb{R}^1 = \mathbb{R}^3$



② Horizontal slices by planes $z = c$ are L_c.



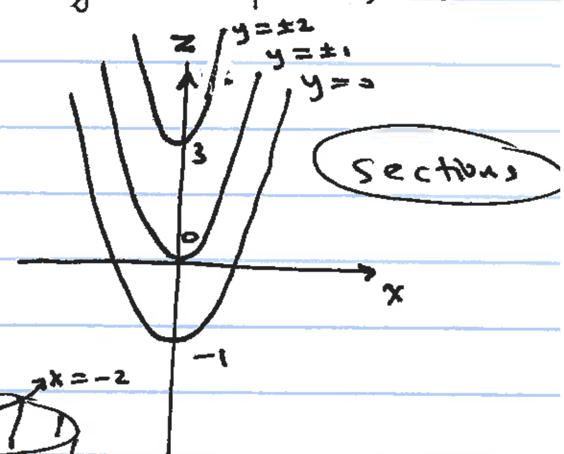
Sections: are obtained by restricting some domain variables to constants:

Continue

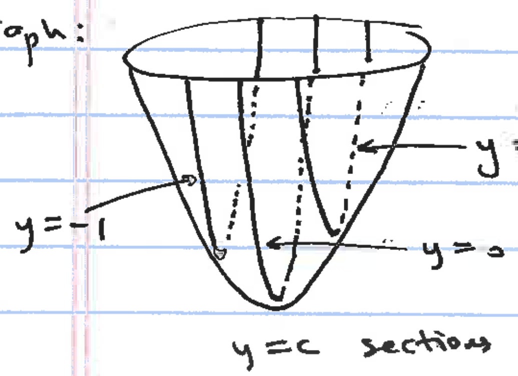
Ex 3 $f(x,y) = x^2+y^2-1$

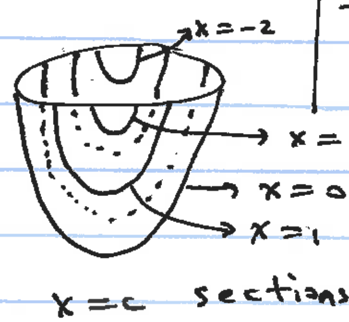
$x=0$	$f(0,y) = y^2-1$	or	$y=0$	$f(x,0) = x^2-1$
$x=\pm 1$	$f(\pm 1,y) = y^2$		$y=\pm 1$	$f(x,1) = x^2$
$x=\pm 2$	$f(2,y) = 3+y^2$.		$y=\pm 2$	$f(x,2) = 3+x^2$

These are the slices of the graph with the planes $x = -2, -1, 0, 1, 2$

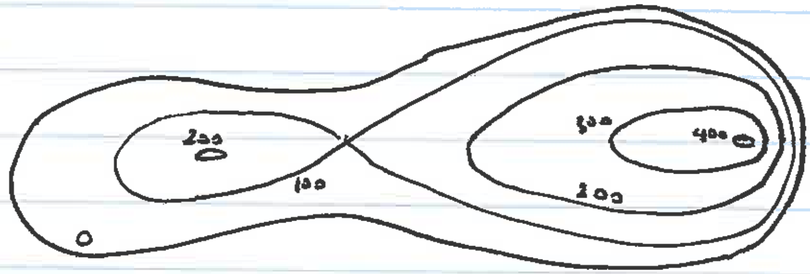


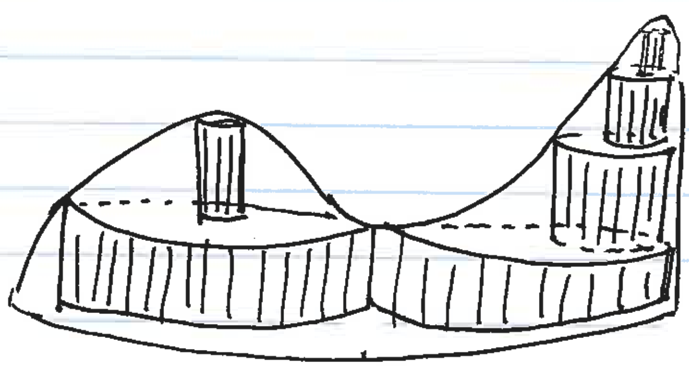
Graph:





Ex 4 Topographical map of an island

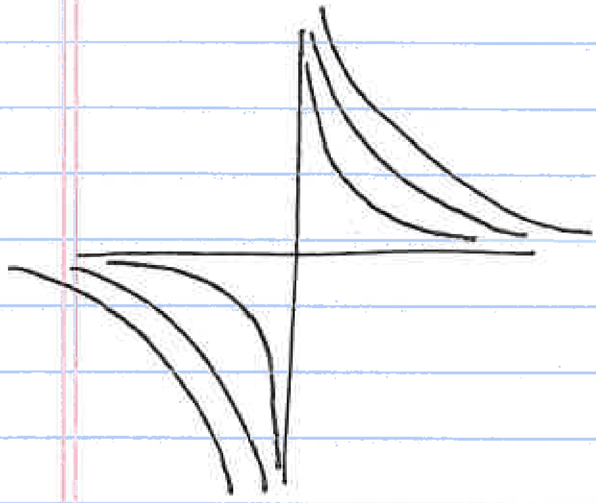




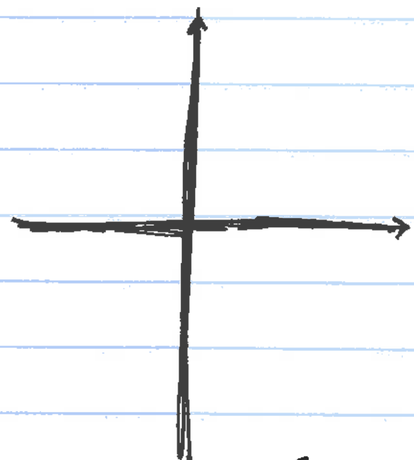
Example 5

$$f(x,y) = xy : \mathbb{R}^2 \to \mathbb{R}^1$$

Level curves $L_c = \{(x,y) \mid f(x,y) = xy = c\}$.

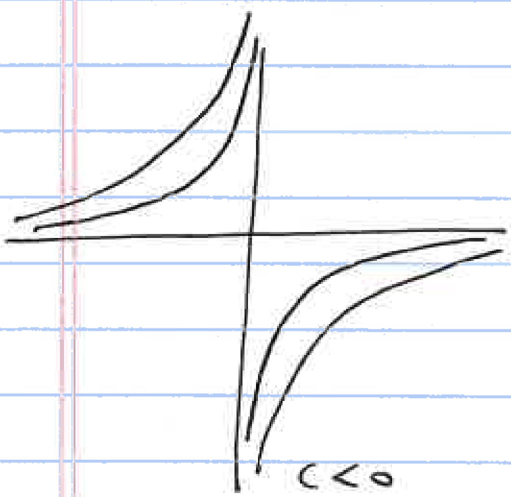


$c > 0$



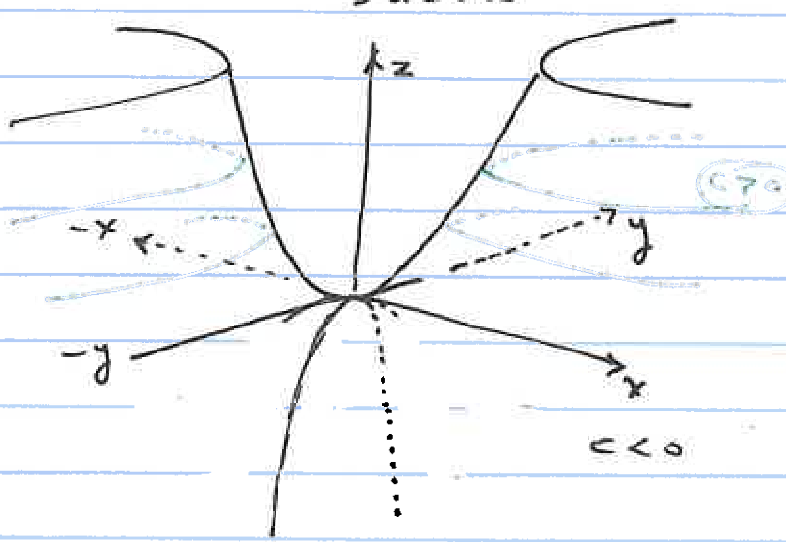
$c = 0 = xy$

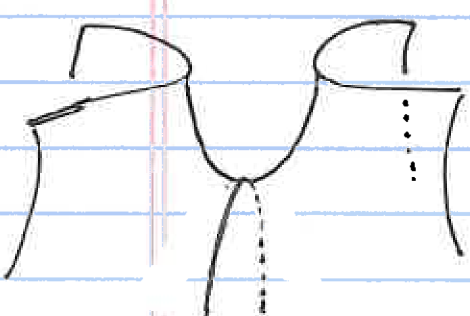
$\iff \underbrace{x = 0 \text{ or } y = 0}_{\text{union of } x \text{ \& } y \text{ axes}}$



$c < 0$

Saddle





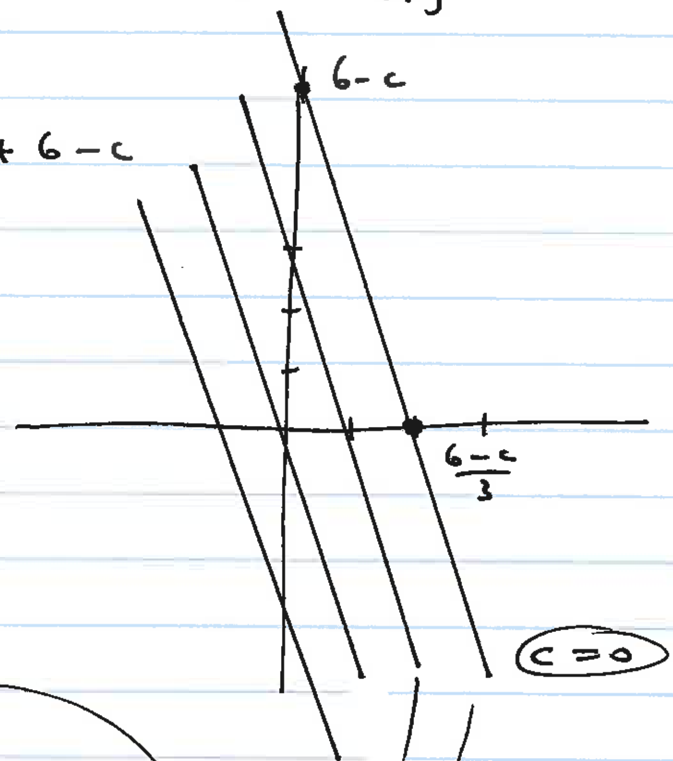
Example 6

$$h(x,y) = 6 - 3x - y \;:\; \underset{x,y}{\mathbb{R}^2} \longrightarrow \underset{z.}{\mathbb{R}}$$

$$6 - 3x - y = c$$

$$y = -3x + 6 - c$$

Level sets



Sections.

$x = 0$ (yz plane)
$z = h(0,y) = 6 - y$

$y = 0$ (xz plane)
$z = h(x,0) = 6 - 3x.$