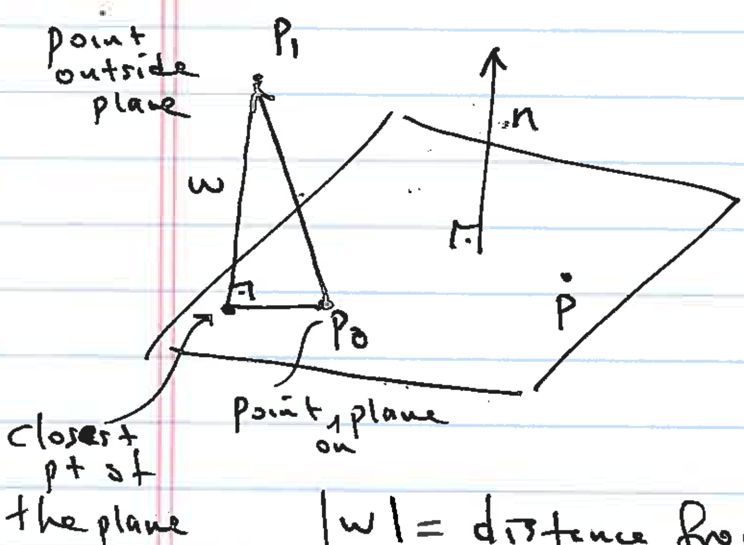
Sept 7 (1)

(1.5) To finish

Example distance from a point to a plane given by a closed eq^n.



$$\underbrace{\vec{n}\cdot(p - p_0) = 0.}_{\text{eq}^n \text{ of plane}}$$

$$w = \text{proj}_n(p_1 - p_0)$$

$|w|$ = distance from p_1 to the plane $n\cdot(p-p_0)=0$

$$|w| = \left|\text{proj}_n(p_1-p_0)\right|$$

$$= \left|\frac{(p_1-p_0)\cdot n}{n\cdot n}\, n\right|$$

$$= \frac{|(p_1-p_0)\cdot n|}{\|n\|^2}\,\|n\|$$

$$= \frac{|(p_1-p_0)\cdot n|}{\|n\|}$$

page 48

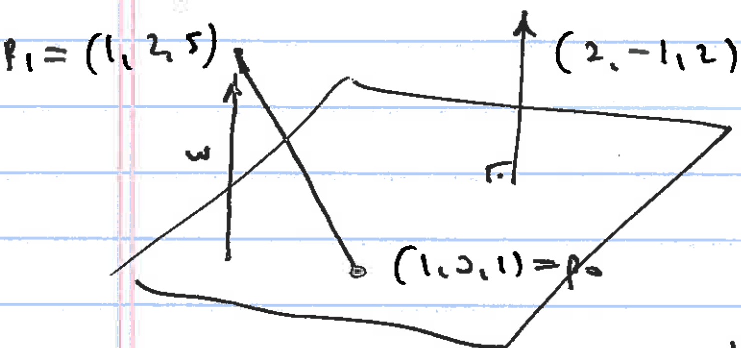
Exc #37: whole page gives the solution of #37.

Ex $(*)\ 2x - y + 2z = 4$
$P_1 = (1,2,5)$] Find distance between pt & plane

$n = (2,-1,2)$

$P_0 = (1,0,1)$ (Find by inspection of $(*)$)

distance



$P_1 - P_0 = (1,2,5) - (1,0,1)$
$= (0,2,4)$

$$w = \mathrm{proj}_{(2,-1,2)}(0,2,4)$$

$$= \frac{(2,-1,2)\cdot(0,2,4)}{(2,-1,2)\cdot(2,-1,2)} \cdot (2,-1,2)$$

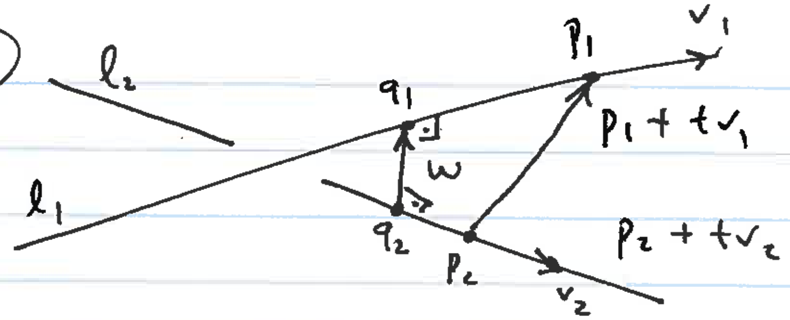
$$= \frac{0-2+8}{4+1+4}(2,-1,2)$$

$$w = \frac{2}{3}(2,-1,2)$$

$$|w| = \frac{2}{3}\cdot 3 = 2$$

Distance between skew lines.

Ex



$q_1 \in \ell_1$

$q_2 \in \ell_2$

closest pts of ℓ_1 & ℓ_2

We do not know q_1 and q_2.

Exc 39 p. 48

$w \perp v_1$

$w \perp v_2$ $\qquad \operatorname{proj}_{v_1 \times v_2}(P_1 - P_2) = w$

$w \parallel v_1 \times v_2$.

$|w| =$ distance between ℓ_1 & ℓ_2

$$|w| = \left|\operatorname{proj}_{v_1 \times v_2}(p_1 - p_2)\right| = \frac{|(p_1 - p_2)\cdot v_1 \times v_2|}{\|v_1 \times v_2\|}$$

Ex Find the distance between ℓ_1 & ℓ_2.

$$\ell_1 \begin{cases} x = 1 - t \\ y = 2 + t \\ z = 3 + 4t \end{cases} \qquad \ell_2 \begin{cases} x = 1 + 2t \\ y = 5 - t \\ z = 0 \end{cases}$$

$$\underbrace{(1,2,3)}_{p_1} + t\underbrace{(-1,1,4)}_{v_1} \qquad \underbrace{(1,5,0)}_{p_2} + t\underbrace{(2,-1,0)}_{v_2}$$

$$v_1 \times v_2 = \begin{vmatrix} i & j & k \\ -1 & 1 & 4 \\ 2 & -1 & 0 \end{vmatrix} = (4, 8, -1)$$

$$p_1 - p_2 = (1,2,3) - (1,5,0) = (0,-3,3)$$

$$w = \text{proj}_{v_1 \times v_2}(p_1 - p_2) = \text{proj}_{(4,8,-1)}(0,-3,3)$$

$$= \frac{(4,8,-1)\cdot(0,-3,3)}{(4,8,-1)\cdot(4,8,-1)}(4,8,-1)$$

$$= \frac{0-24-3}{16+64+1}(4,8,-1)$$

$$= \frac{-27}{81}(4,8,-1) = -\frac{1}{3}(4,8,-1)$$

$\|w\| = \frac{1}{3}\cdot 9 = 3 =$ distance between l_1 & l_2.

1.6 will not cover
1.7 will cover later

2.1

<u>Defn</u> A function $f: X \to Y$ is a rule of assignment that associates to each element $x \in X$ a unique element $f(x) \in Y$.

X = domain
Y = codomain
$f(X)$ = range f = $\{ f(x) \mid x \in X \}$.
$= \{ y \in Y \mid y = f(x) \text{ for some } x \in X \}$.

- A function f is called onto (surjective) if range f = codomain of f (Y = range f for $f: X \to Y$)

- A function f is called one-to-one (injective) if $\forall x_1, x_2 \in X$ $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
($\Leftrightarrow$ $\forall x_1, x_2 \in X$ $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$)

n = number of variables
m = number of components.

We are interested in $X \subseteq \mathbb{R}^n$, $Y \subseteq \mathbb{R}^m$
$f: X \to Y$.

<u>Example 1</u> $f(x,y) = x^2 + y^2 + 4$
$f: \underset{x,y}{\mathbb{R}^2} \longrightarrow \mathbb{R}^1$ 1 component

Domain $f = \mathbb{R}^2$, Range $= [4, \infty)$

Example 2

$$g(\underbrace{x,y,z}_{3\,=\,\#\text{ variables}}) = (x+y,\ x^2+y^3-z)$$

each called a component function

$\#$ components $= 2$

$$g: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

Is g 1-1? No $\left.\begin{array}{l} g(-1,1,2) = (0,0) \\ g(1,-1,0) = (0,0) \end{array}\right\}$ $(-1,1,2) \neq (1,-1,0)$

Is g onto $\mathbb{R}^2$? YES $\forall (a,b) \in \mathbb{R}^2$

$$g(0, a, a^3-b) = (a,b)$$

End of 9/7/16

GRAPHS: $f: X \subseteq \mathbb{R}^n \longrightarrow Y \subseteq \mathbb{R}^m$

(Explicit) Graph $f = \{ (\vec{x}, \overrightarrow{f(x)}) \in \mathbb{R}^n \times \mathbb{R}^m \mid \vec{x} \in X \}$
$\subseteq \mathbb{R}^n \times \mathbb{R}^m$

$n = \#$ variables
$m = \#$ components.

Parametric graph $f = \{ f(\vec{x}) \in \mathbb{R}^m \mid x \in X \subseteq \mathbb{R}^n \}$
$\subseteq \mathbb{R}^m$

Implicit graph: $\forall b \in Y$
$\{ x \in X \subseteq \mathbb{R}^n \mid f(x) = b \}$

Also known as

Contour graphs / Inverse Images
Level sets /