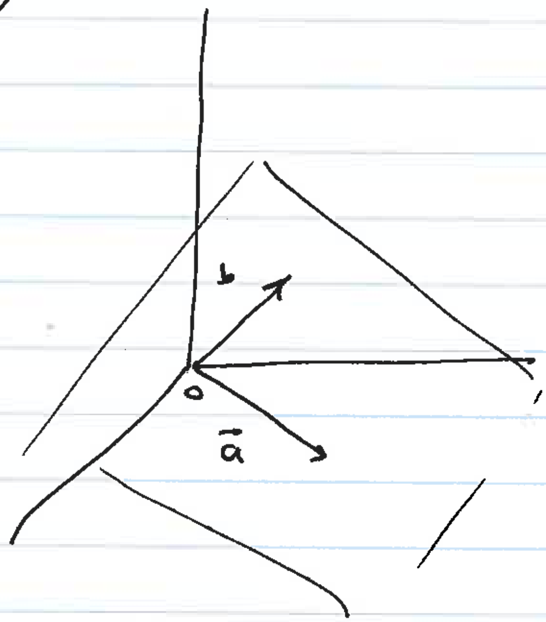
1.5

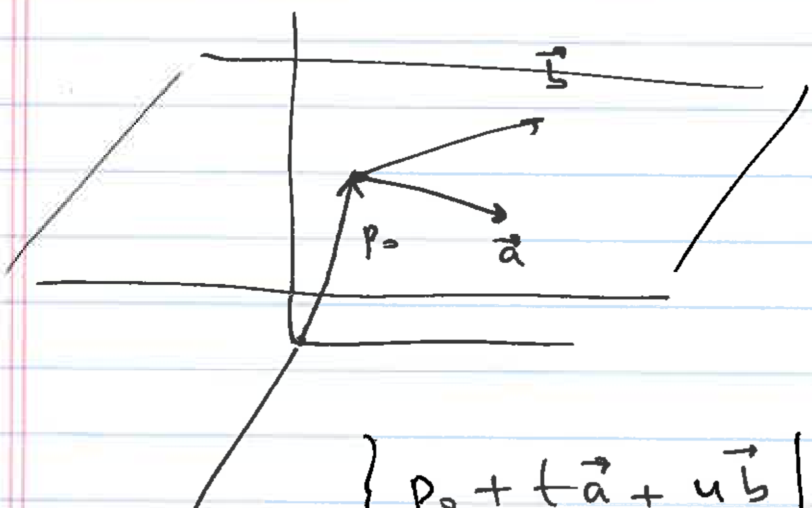


Given $\vec{a}, \vec{b} \in \mathbb{R}^n$

$\{ t\vec{a} + u\vec{b} \mid t, u \in \mathbb{R} \}$

$\vec{a} \neq 0$, $\vec{b} \neq 0$, $\vec{a}$ not $\vec{b}$ parallel

Subspace spanned by $\vec{a}, \vec{b}$ is a plane thru $\vec{0}, \vec{a}, \vec{b}$.



$\{ p_0 + t\vec{a} + u\vec{b} \mid t, u \in \mathbb{R} \}$

is a plane thru p_0, and parallel to $\vec{a}$ and $\vec{b}$, provided that $\vec{a} \neq 0$, $\vec{b} \neq 0$, $\vec{a}$ not parallel to $\vec{b}$.

Recall: $\vec{a} \neq 0$, $\vec{b} \neq 0$ if $\exists \lambda \in \mathbb{R}$, $\lambda \neq 0$ $\vec{a} = \lambda \vec{b}$ then $\vec{a}$ & $\vec{b}$ are parallel, & vice versa.

Given, $\vec{a}, \vec{b} \neq 0$:

$$\begin{cases} a \parallel b \iff \exists \lambda \in \mathbb{R} - \{0\} \quad \vec{a} = \lambda \vec{b} \\ a \not\parallel b \iff \forall \lambda \in \mathbb{R} - \{0\} \quad \vec{a} \neq \lambda \vec{b}. \end{cases}$$

Examples:

<u>Exc # 18 p47</u>

$$f(s,t) = (2,9,-4) + t(-8,2,5) + s(3,-4,-2)$$
$$= (x,y,z)$$

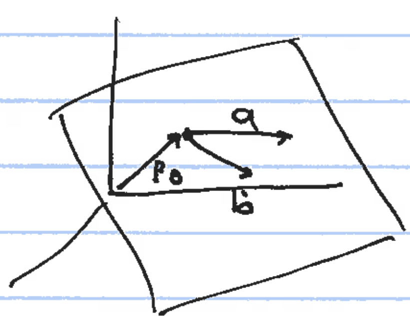
$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$
$$s,t \qquad\qquad x,y,z.$$

vector parametric representation

This plane is the parametric graph of $(x,y,z) = f(s,t)$

$$\left.\begin{aligned} x &= 2 - 8t + 3s \\ y &= 9 + 2t - 4s \\ z &= -4 + 5t - 2s \end{aligned}\right\} \text{scalar parametric representation.}$$

Not from the book. (b) Find a closed equation for the same plane



$P_0 = (2,9,-4)$

$\vec{a} = (-8,2,5)$

$\vec{b} = (3,-4,-2)$

Can take $n = \begin{vmatrix} i & j & k \\ -8 & 2 & 5 \\ 3 & -4 & -2 \end{vmatrix}$

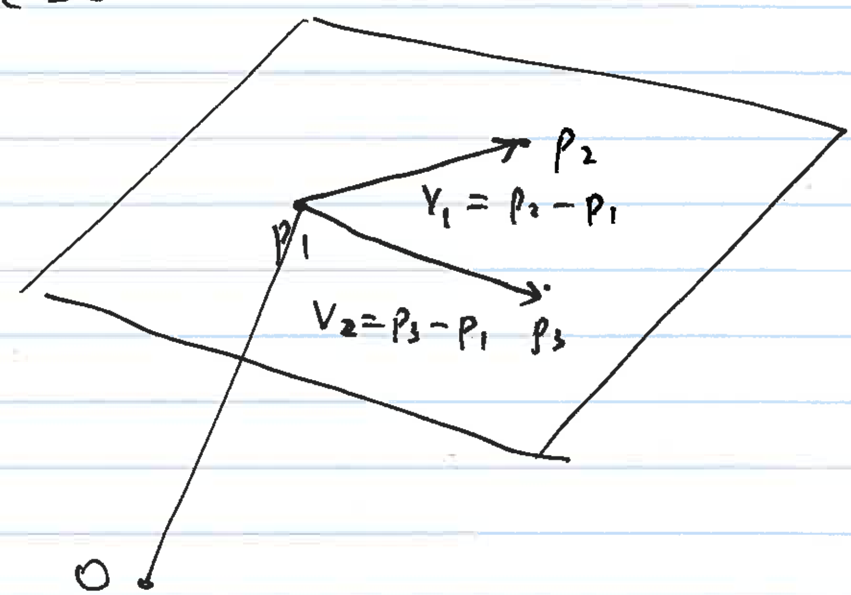
$= (16, -1, 26)$

Closed Eq.

$$16x - y + 26z = D$$
$$= (16, -1, 26) \cdot (2, 9, -4)$$
$$= 32 - 9 - 104$$

Ans: $16x - y + 26z = -81$

Exc #20 Equation of plane thru 3 pts.



$(0, 2, 1)$
$(7, -1, 5)$
$(-1, 3, 0)$

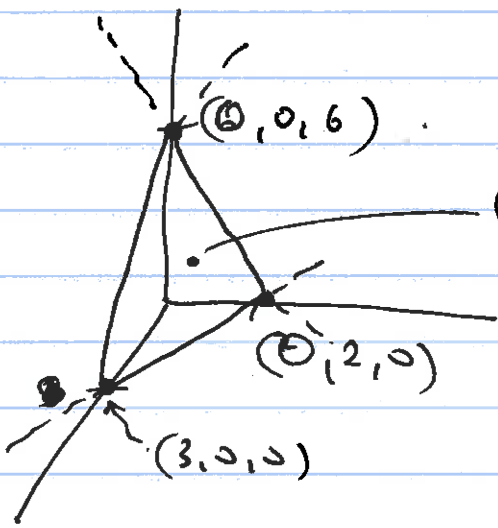
$$p_1 + sv_1 + tv_2$$

$$f(s,t) = (0, 2, 1) + s\underbrace{(7, -3, 4)}_{(7,-1,5)-(0,2,1)} + t\underbrace{(-1, 1, -1)}_{(-1,3,0)-(0,2,1)}$$

Ex.

$$2x + 3y + z = 6$$ closed eqn.

Find a parametric rep.



(1,1,1) is on the plane

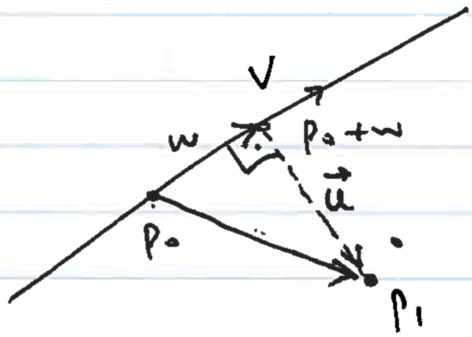
Check

$2 \cdot 1 + 3 \cdot 1 + 1 = 6$ ✓

$$f(s,t) = (1,1,1) + t(-3,2,0) + s(-3,0,6)$$

Distance

(A) Distance from a pt to a parametric line.



$\ell(t) = p_0 + tv$

$$\text{proj}_v(p_1 - p_0) = w$$

$$\text{dist} = |\underbrace{p_1 - (p_0 + w)}_{\vec{u}}|$$

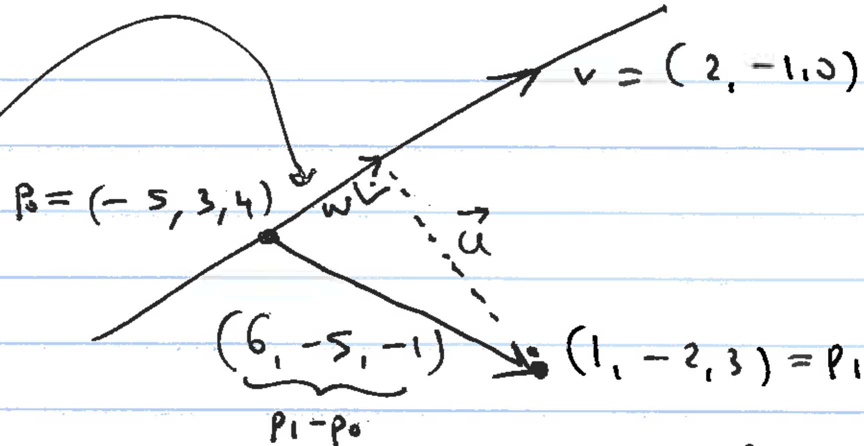
p48 <u>Exc #24</u>

pt $(1,-2,3)$ "p_1" outside line

line $\begin{cases} x = 2t - 5 \\ y = 3 - t \\ z = 4 \end{cases}$ $\longrightarrow$ pt $= (-5,3,4)$, $v = (2,-1,0)$

pt on the line $= p_0$

$$p_1 - p_0 = (1,-2,3) - (-5,3,4)$$
$$= (6,-5,-1)$$



$$\text{proj}_{(2,-1,0)}(6,-5,-1) = \frac{(6,-5,-1)\cdot(2,-1,0)}{(2,-1,0)\cdot(2,-1,0)}(2,-1,0)$$

$$= \frac{12+5+0}{5}(2,-1,0)$$

$$= \frac{17}{5}(2,-1,0) = w$$

$$p_1 - p_0 - w = \vec{u} = (6,-5,-1) - \frac{17}{5}(2,-1,0)$$

$$= \left(6 - \frac{34}{5}, -5 + \frac{17}{5}, -1\right)$$

$$= \left(-\frac{4}{5}, \frac{-8}{5}, -1\right) \checkmark$$ (OK, matches my notes.)

distance from p_1 to ℓ $= \|u\| = \sqrt{\frac{16}{25} + \frac{64}{25} + 1} = \sqrt{\frac{105}{25}} = \frac{\sqrt{105}}{5}$