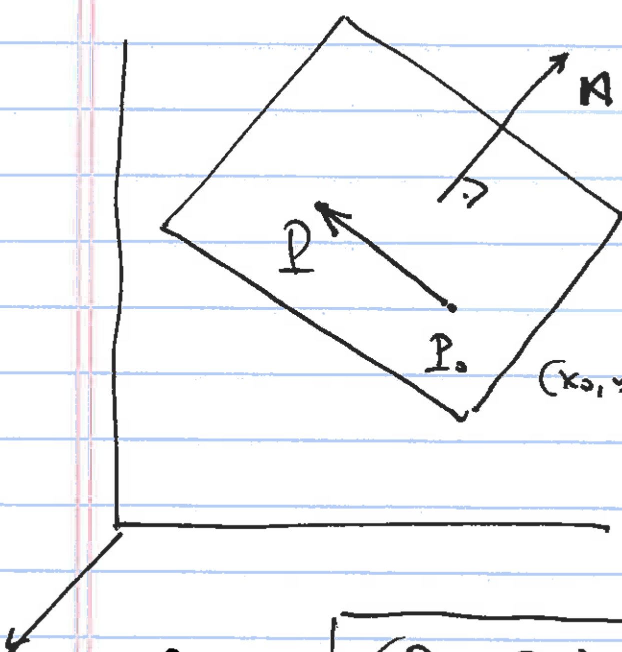
Sept 1, 2016

1.5

Lines : parametric eqns in $\mathbb{R}^n$

: symmetric eqns in $\mathbb{R}^3$

PLANES Closed equations.



$\vec{n} = (A, B, C) \neq \vec{0}$ perpendicular to the plane

P_0 a particular pt of the plane

$P_0 = (x_0, y_0, z_0)$

$P = (x, y, z)$ is a generic pt of the plane

all equivalent

$$\boxed{(P - P_0) \cdot \vec{n} = 0.}$$

$$\big((x, y, z) - (x_0, y_0, z_0)\big) \cdot (A, B, C) = 0$$

$$(x - x_0, y - y_0, z - z_0) \cdot (A, B, C) = 0$$

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$Ax + By + Cz = Ax_0 + By_0 + Cz_0 = D.$$

$$\boxed{Ax + By + Cz = D}$$

1.5 Ex #2

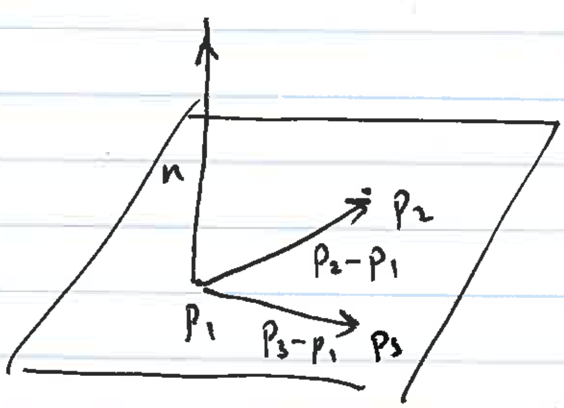
Plane containing $(9,5,-1)$
$\perp$ $\vec{i}-2\vec{k}$

$$\left((x,y,z)-(9,5,-1)\right)\cdot \underbrace{(i-2\vec{k})}_{(1,0,-2)} = 0$$

$$1\cdot x + 0y - 2z = (9,5,-1)\cdot(1,0,-2)$$

$$x - 2z = 11$$

Ex Find an equation for the plane containing $(1,2,3) = p_1$
$(1,1,1) = p_2$
$(-1,1,5) = p_3$



Can take
$(p_3 - p_1)\times(p_2 - p_1) = n$

$$p_3 - p_1 = (-1,1,5) - (1,2,3) = (-2,-1,2)$$
$$p_2 - p_1 = (1,1,1) - (1,2,3) = (0,-1,-2)$$

$$\begin{vmatrix} i & j & k \\ -2 & -1 & 2 \\ 0 & -1 & -2 \end{vmatrix} = (4, -4, 2)$$

$$4x - 4y + 2z = D$$

$(1,1,1)$ on the plane

$$4 \cdot 1 - 4 \cdot 1 + 2 \cdot 1 = D = 2.$$

$$4x - 4y + 2z = 2$$

$$\boxed{2x - 2y + z = 1}$$

$(1,1,1)$ ✓
$(1,2,3)$ ✓
$(-1,1,5)$ ✓

Ex Find an equation of a plane through $(1,-2,6)$ & parallel to $3x + 4y - z = 10$

Answer:

$$3x + 4y - z = 3 \cdot 1 + 4(-2) - 1 \cdot 6$$

$$= 3 - 8 - 6$$

$$3x + 4y - z = -11$$

Ex Find a parametric equation for the line of intersection of the 2 planes

- $x + 2y + 3z = 4$
- $2x + 6y + 10z = 2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 6 & 10 & 2 \end{array}\right] \xrightarrow[R_2 - 2R_1]{} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 2 & 4 & -6 \end{array}\right]$$

$$\xrightarrow[R_1 - R_2 \to R_1]{} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 10 \\ 0 & 2 & 4 & -6 \end{array}\right] \xrightarrow[\frac{1}{2}R_2]{} \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 10 \\ 0 & 1 & 2 & -3 \end{array}\right] \\ \qquad \uparrow t = z \end{array}.$$

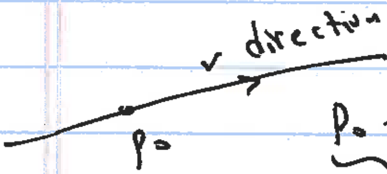
$$\left.\begin{aligned} x - t &= 10 \\ y + 2t &= -3 \\ z &= t \end{aligned}\right\} \qquad \left.\begin{aligned} x &= t + 10 \\ y &= -2t - 3 \\ z &= t \end{aligned}\right\}$$

$$r(t) = (10, -3, 0) + t(1, -2, 1)$$

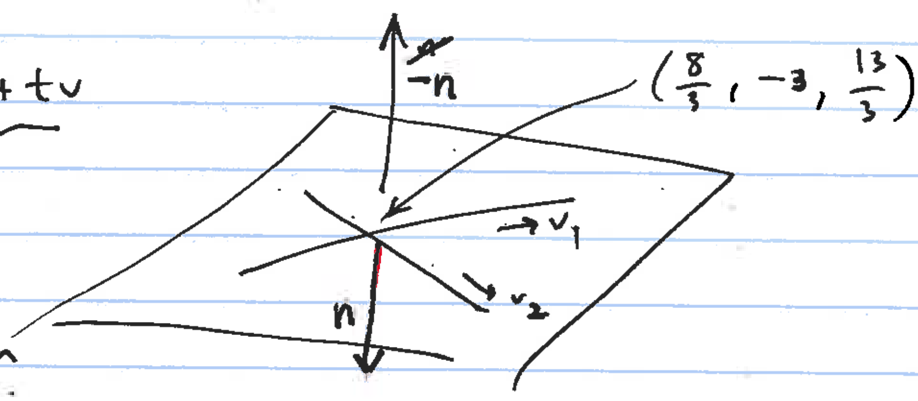
Exc #12 Plane containing 2 given intersecting lines

ℓ_1 $\left.\begin{array}{l} x = t+2 \\ y = 3t-5 \\ z = 5t+1 \end{array}\right\}$ direction of ℓ_1 $= (1,3,5) = v_1$

ℓ_2 $\left.\begin{array}{l} x = 5-t \\ y = 3t-10 \\ z = 9-2t \end{array}\right\}$ direction of ℓ_2 $= (-1,3,-2) = v_2$



$\underline{p_0 + tv}$



Recall RHR for the direction of n.

$$n = v_1 \times v_2 = \begin{vmatrix} i & j & k \\ 1 & 3 & 5 \\ -1 & 3 & -2 \end{vmatrix} = (-21, -3, 6)$$

Plane: $-21x - 3y + 6z = (2,-5,1)\cdot(-21,-3,6)$

take $t=0$ → $(2,-5,1)$ is on ℓ_1; so: $(2,-5,1)$ is on the plane

$$-21x - 3y + 6z = -42 + 15 + 6 = -21$$

$$\boxed{7x + y - 2z = 7}$$