(1.2) **Exercises:** Aug 26, 2016 (1)

Exc # 42 p 17.

Find intersection of lines

$$x = 2t + 3 \qquad x = 15 - 7t$$
$$y = 3t + 3 \qquad y = t - 2$$
$$z = 2t + 1 \qquad z = 3t - 7.$$

Soln

$$\begin{cases} 2t_1 + 3 = x = 15 - 7t_2 \\ 3t_1 + 3 = y = t_2 - 2 \\ 2t_1 + 1 = z = 3t_2 - 7. \end{cases}$$

simplify

solve

$$\begin{cases} 2t_1 + 7t_2 = 12 \\ 3t_1 - t_2 = -5 \\ 2t_1 - 3t_2 = -8 \end{cases}$$

possibilities:

(1) if only many sol^ns ⟹ equal lines with possibly diff. parameterizations

(2) unique soln. 1 pt of intersection

(3) No sol^n.

Skew lines | parallel lines

non-parallel directions.

but ta direction a parallel direction

Solve PTO

Ex 42. p17 Continues

$$2t_1 + 7t_2 = 12$$
$$3t_1 - t_2 = -5$$
$$2t_1 - 3t_2 = -8$$

<u>NOT</u> required to use Row Reduction

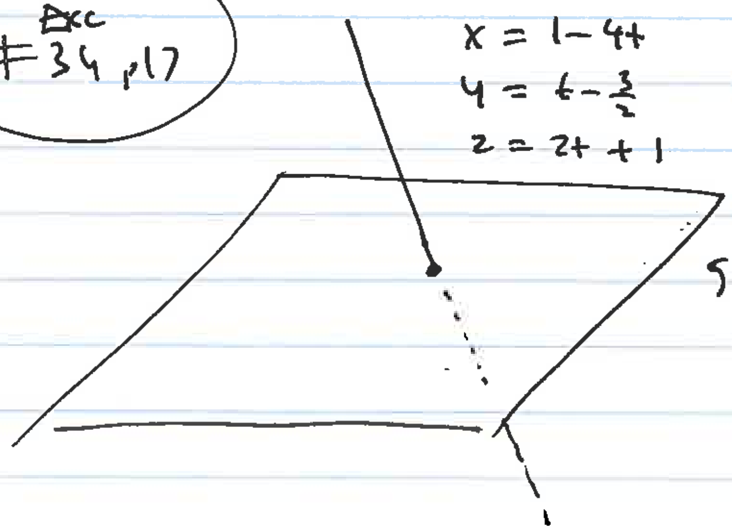
$$\left[\begin{array}{cc|c} 2 & 7 & 12 \\ 3 & -1 & -5 \\ 2 & -3 & -8 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 2 & 7 & 12 \\ 3 & -1 & -5 \\ 0 & -10 & -20 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cc|c} 2 & 7 & 12 \\ 3 & -1 & -5 \\ 0 & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 2 & 0 & -2 \\ 3 & 0 & -3 \\ 0 & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left.\begin{aligned} t_1 &= -1 \\ t_2 &= 2 \end{aligned}\right\}$$

pt of intersection $(1, 0, -1)$

Exc #34, p17



$x = 1 - 4t$
$y = t - \frac{3}{2}$
$z = 2t + 1$

Pt of intersection of a plane & parametric line

$$5x - 2y + z = 1$$

$$5(1 - 4t) - 2\left(t - \tfrac{3}{2}\right) + (2t + 1) = 1$$

$$5 - 20t - \cancel{2t} + 3 + \cancel{2t} + \cancel{1} = \cancel{1}$$

$$8 - 20t = 0 \qquad t = \frac{2}{5}$$

$\left(-\frac{3}{5}, -\frac{11}{10}, \frac{9}{5}\right)$
$\|$
Pt of intersect
$\left(1 - \frac{8}{5}, \frac{2}{5} - \frac{3}{2}, \frac{9}{5}\right)$

In $\mathbb{R}^3$ Symmetric form of a line: $a_1, a_2, a_3 \in \mathbb{R}$, $b_1, b_2, b_3 \in \mathbb{R}$

$$\underbrace{\frac{x-b_1}{a_1} = \frac{y-b_2}{a_2}}_{\text{plane}} = \frac{z-b_3}{a_3} \quad \text{if each } a_i \neq 0$$

plane

All 3 equal $\Longrightarrow$ line of intersection of two planes.

How do parametric representation and symmetric form relate?

Ex

$r(t) = (1, 2, 5) + t(7, -1, 3)$ Vector parametric

$$\left.\begin{aligned} x &= 1 + 7t \\ y &= 2 - t \\ z &= 5 + 3t \end{aligned}\right\} \text{ Scalar parametric}$$

$$t = \frac{x-1}{7} = \frac{y-2}{-1} = \frac{z-5}{3} \qquad \text{Symmetric}$$

1.3

<u>Defn</u> $\forall\ \vec{a} = (a_1, a_2 \dots a_n) \in \mathbb{R}^n$
$\vec{b} = (b_1, b_2 \dots b_n) \in \mathbb{R}^n$

we define $$\vec{a} \cdot \vec{b} = \sum_{i=1}^{n} a_i b_i$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$$

Ex

$$(2,3) \cdot (-1,7) = 2 \cdot -1 + 3 \cdot 7 = 19$$

$$(\vec{i} + 5\vec{j} + 7\vec{k}) \cdot (3\vec{i} + 6\vec{k}) = 3 + 0 + 42$$
$$= 45.$$

<u>Properties</u> $\vec{a} \cdot \vec{a} \geq 0$

$$\vec{a} \cdot \vec{a} = a_1^2 + a_2^2 + \dots + a_n^2 \geq 0$$

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

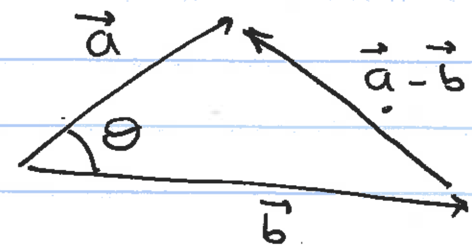
$$(k\vec{a}) \cdot \vec{b} = k(\vec{a} \cdot \vec{b})$$

$$||a||^2 = \vec{a} \cdot \vec{a}$$

Prop. Let $\vec{a}, \vec{b}$ be vectors in $\mathbb{R}^n$, $\vec{a} \neq 0$, $\vec{b} \neq 0$.

Then $a \cdot b = \cos\theta \cdot \|a\| \cdot \|b\|$ where θ is the angle between a & b.

Proof:



Law of cosines

$$\|\vec{a}-\vec{b}\|^2 = \|\vec{a}\|^2 + \|\vec{b}\|^2 - 2\|\vec{a}\|\,\|\vec{b}\|\cos\theta.$$

$$\|$$

$$(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$\|$$

$$\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$$

$$\|$$

$$\|a\|^2 - 2\,a\cdot b + \|b\|^2$$

Conclusion $a \cdot b = \|a\|\,\|b\|\cos\theta.$

Consequences: if $\vec{a}, \vec{b} \neq 0$ then

① $$\cos\theta = \frac{a \cdot b}{\|a\| \, \|b\|}$$ where

θ is the angle between $\vec{a}$ & $\vec{b}$.

② $$|a \cdot b| \leq \|a\| \, \|b\| \quad (\text{since } |\cos\theta| \leq 1)$$

③ $$a \cdot b = 0 \iff \theta = \frac{\pi}{2}$$
(Assuming $\vec{a}, \vec{b} \neq 0$) $\quad \vec{a} \perp \vec{b}$.

Exc.

1.3 #8 Find angle between $\vec{a} = (-1, 2)$
$\vec{b} = (3, 1)$

$\|a\| = \sqrt{5}$
$\|b\| = \sqrt{10}$
$a \cdot b = -3 + 2 = -1$

$$\cos\theta = \frac{a \cdot b}{\|a\| \, \|b\|} = \frac{-1}{\sqrt{5}\sqrt{10}} = \frac{-1}{5\sqrt{2}}.$$

$$\theta = \cos^{-1}\left(\frac{-1}{5\sqrt{2}}\right)$$