Aug 25, 2016

Announcement:

Required

- Quiz 1 — Sept 9
- Quiz 2 — Sept 23

MT 1 — Sept 30

- Quiz 3 — Oct 14
- Quiz 4 — Oct 28

MT 2 — Nov 11

- Quiz 5 — Dec 2

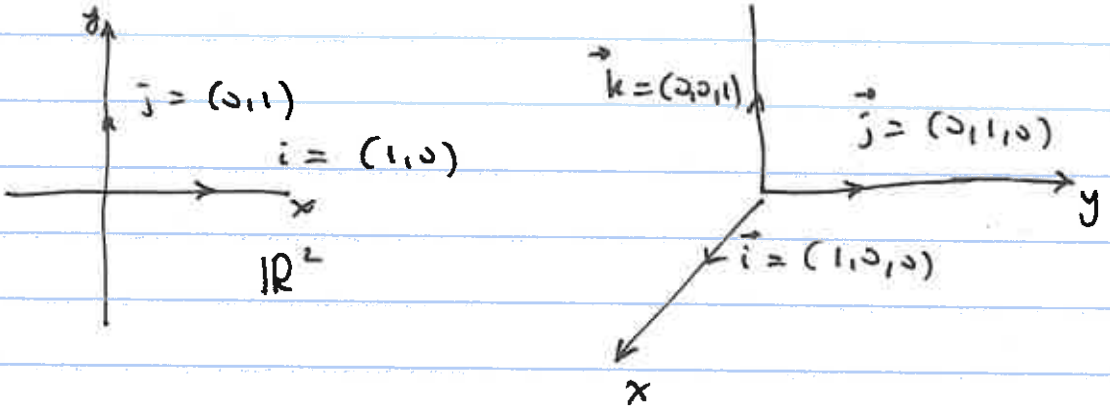
optional

- (Quiz 6) — Dec 9

If needed or requested by students.

1.2

$\vec{i}, \vec{j}, \vec{k}$



Ex $3\vec{i} + 4\vec{j} + 7\vec{k} = (3, 4, 7)$

$2\vec{i} - 3\vec{j} = (2, -3)$ in $\mathbb{R}^2$

$2i - 3j = (2, -3, 0)$ in $\mathbb{R}^3$

$\mathbb{R}^3$ $\{\vec{i}, \vec{j}, \vec{k}\}$ $\Rightarrow$ a basis of $\mathbb{R}^3$

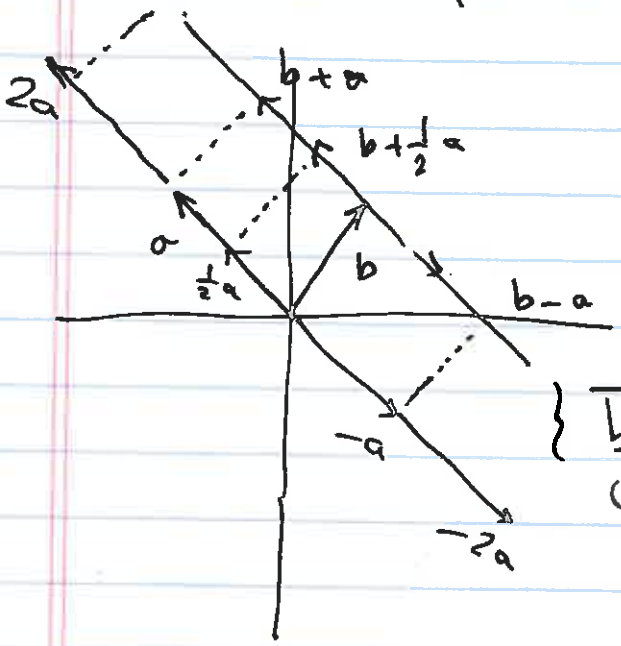
Spans $\mathbb{R}^3$
Every vector in $\mathbb{R}^3$ can be written as a linear combination of i, j, k.

linear Indep.
$\Downarrow$
This linear combination is unique $\forall \vec{v}$ in $\mathbb{R}^3$

Caution $(x_1, x_2 \cdots x_n)$ can be used for pts or vector.

Book is not consistent
$\left\{ \begin{array}{l} P(x_1 \cdots x_n) \text{ used for pts.} \\ a\vec{i} + b\vec{j} + c\vec{k} \text{ used for vectors} \end{array} \right.$

Parametric Equations of lines:



Let $\vec{a}, \vec{b}$ fixed vectors
$\vec{a} \neq 0$

$$\{\vec{b} + t\vec{a} \mid t \in \mathbb{R}\}$$

line passing thru $\vec{b}$ parallel to $\vec{a}$.

Caution only the tips of the vectors

Prop $\vec{r}(t) = \vec{b} + t\vec{a}$ represents a line (parametrically) through b & parallel to $\vec{a}$.

Exc # 14 p 16 | Line through $(12, -2, 0)$ // $(5i - 12j + k)$

$$(x, y, z) = \vec{r}(t) = (12, -2, 0) + t(5, -12, 1)$$
$$= (12 + 5t, -2 - 12t, 0 + t)$$

vector parametric form.

OR

$$\left.\begin{aligned} x &= 12 + 5t \\ y &= -2 - 12t \\ z &= 0 + t \end{aligned}\right\}$$ scalar parametric form.

Exc #16 p 16

Line in $\mathbb{R}^3$ through $(2,1,2) = P$, $(3,-1,5) = Q$ $\Big\}$ $Q - P = (1,-2,3)$

Each is a correct answer.

$$\begin{cases} r(t) = (2,1,2) + t(1,-2,3) \\ r(t) = (3,-1,5) + t(1,-2,3) \\ r(t) = (3,-1,5) + t(-1,2,-3) \\ r(t) = (3,-1,5) + t(10,-20,30) \end{cases}$$

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* IMPORTANT: *

Let $f : A \longrightarrow B$ be a function.

domain — codomain

Graph
Full graph
Explicit Graph

$$G = \{(a,b) \mid \forall a \in A,\ b = f(a)\} \subseteq A \times B.$$

domain x codomain

Implicit Graph for a given $b \in B$.

$$H = \{a \in A \mid f(a) = b\}$$

$H \subseteq A$ domain

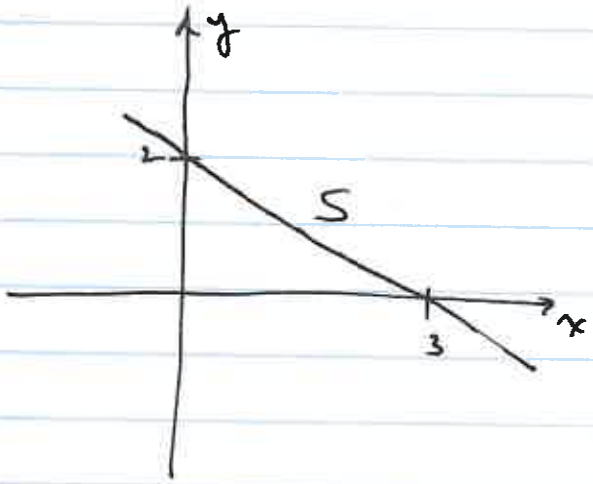
Parametric Graph.

$$L = \{f(a) \in B \mid a \in A\}$$

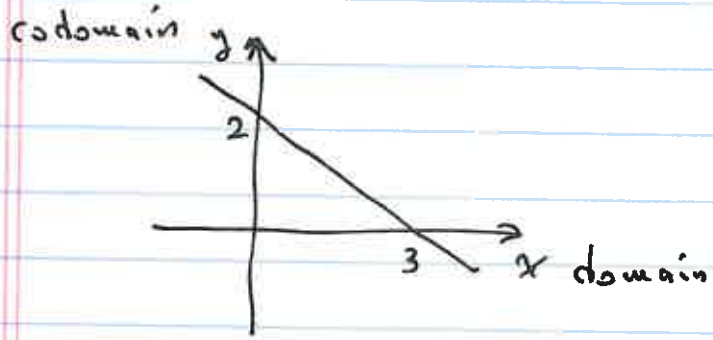
$L \subseteq B$ codomain.

Ex (a) $S = \{(x,y) \mid 2x + 3y = 6\}$. Set

No function here.



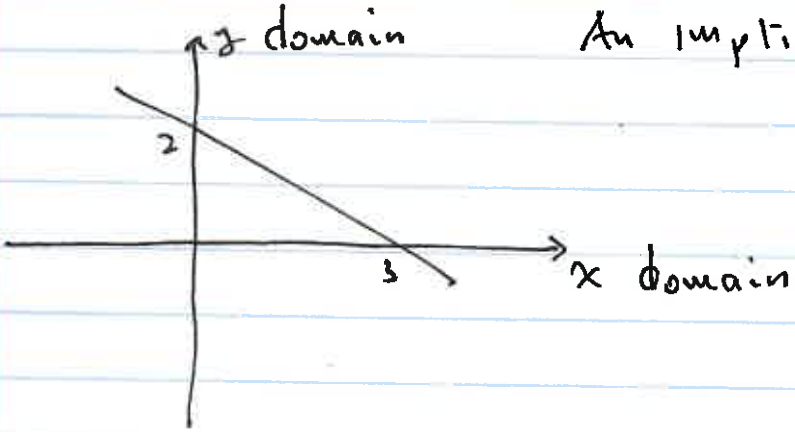
(i) $y = f(x) = \frac{6-2x}{3} = 2 - \frac{2}{3}x : \underset{x}{\mathbb{R}} \longrightarrow \underset{y}{\mathbb{R}}$



Explicit graph of f

(ii) $F : \underset{x,y}{\mathbb{R}^2} \longrightarrow \underset{z}{\mathbb{R}}$ $\quad F(x,y) = 2x + 3y - 6$

$\{(x,y) \mid F(x,y) = 0\}$

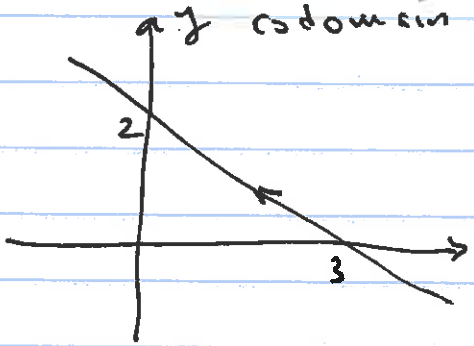


An implicit graph of F

(ii) $r(t): \mathbb{R} \longrightarrow \mathbb{R}^2$

$t \qquad (x,y)$

$t \longrightarrow (3,0) + t(-3,2)$



parametric graph of $\vec{r}(t)$

Exc #28, p 16

* TRICK use t_1, t_2 for different lines

ℓ_1: $x = 2t - 5$, $y = 3t + 2$, $z = 1 - 6t$

ℓ_2: $x = 1 - 2t$, $y = 11 - 3t$, $z = 6t - 17$.

why?

$t_1 = 0$ $x = -5$, $y = 2$, $z = 1$

$t_2 = 3$ $x = -5$, $y = 2$, $z = 1$

$t = 0$ $x = 1$, $y = 11$, $z = -17$

$t_1 = 2$, $t_2 = 1$

$x = -1$, $y = 8$, $z = -11$

$2t_1 - 5 = x = 1 - 2t_2$

$3t_1 + 2 = y = 11 - 3t_2$

$1 - 6t_1 = z = 6t_2 - 17.$

$\Rightarrow \boxed{t_1 = 3 - t_2}$

$1 - 6t_1 = 6t_2 - 17$

$18 - 6t_2 = 6t_1$

$\boxed{3 - t_2 = t_1}$

$2t_1 - 5 = 1 - 2t_2$

$2t_1 = 6 - 2t_2$

$\boxed{t_1 = 3 - t_2}$

$3t_1 + 2 = 11 - 3t_2$

$3t_1 = 9 - 3t_2$

$\boxed{t_1 = 3 - t_2}$