(1.1)

Defn $\mathbb{R}^n = \{ (x_1, x_2 .. x_n) \mid x_i \in \mathbb{R}, \forall i = 1, 2, .. n \}$ ← Given fixed $n \in \mathbb{N}$.

$(x_1, x_2 .. x_n)$: ordered n-tuples, each called a vector.

Defn

$$(a_1, a_2, .. a_n) + (b_1, b_2, .. b_n) = (a_1 + b_1, a_2 + b_2, .., a_n + b_n)$$

$$k \cdot (a_1, a_2, a_3, .. a_n) = (ka_1, ka_2, ka_3, .. ka_n)$$

$$(a_1, a_2, .. a_n) = (b_1, b_2, .. b_n) \iff \forall i \ a_i = b_i$$

(Ex) (Exc 4b)

$$\frac{1}{2}(8, 4, 1) + 2\left(5, -7, \frac{1}{4}\right) = \left(4, 2, \frac{1}{2}\right) + \left(10, -14, \frac{1}{2}\right)$$

$$= (14, -12, 1)$$

Exc (9) p 7

If $(-12, 9, z) + (x, 7, -3) = (2, y, 5)$

What are x, y, z?

Soln

$$(-12 + x, 16, z - 3) = (2, y, 5)$$

Correction → $x = 14$

$$\left.\begin{array}{l} -12 + x = 2 \\ 16 = y \\ z - 3 = 5 \end{array}\right\} \quad \begin{array}{l} x = 14 \\ y = 16 \\ z = 8 \end{array}$$

Caution $(2,3,5) \neq (2,3,5,0)$

$$\underset{\mathbb{R}^3}{(2,3,5)} \neq \underset{\mathbb{R}^4}{(2,3,5,0)}$$

Be Careful about not mixing from different spaces.

$$\underset{\mathbb{R}^2}{(2,3)} + \underset{\mathbb{R}^3}{(5,7,1)} = (7, 10, ??)$$

Oops

You Can't add them

$(\mathbb{R}^n, +, \cdot)$

$$+ : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$
$$(\vec{v}, \vec{w}) \longmapsto \vec{v} + \vec{w}$$

binary operation.

$$\cdot : \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$
$$k, \vec{v} \longmapsto k \cdot \vec{v}$$

Ex $k = 2$, $\vec{v} = (5,-7)$ $\quad k \cdot \vec{v} = (10, -14)$

$\mathbb{R}^1$ $\longleftrightarrow$ 

real numbers algebraic object — 1-1 correspondence — line geometric object

Once

- origin
- unit length
- direction

are chosen.

Every real number corresponds to a unique pt on the line & every point of the line corresponds to a unique real number.