Example  Find the expectation of the random variable whose density is graphed below.

```
0.02
1.1
0.01
1.1
```

20  30  50  70  100

Solution  [Brute Force]

From the graph, it is easy to write down the density of, say $X$, as

$$f(x) = \begin{cases} 
0.02 & 0 \leq x < 20 \\
0.02 - \frac{(x-20)}{10} & 20 \leq x < 30 \\
0.01 & 30 \leq x < 70 \\
0.01 - \frac{(x-70)}{30} & 70 \leq x \leq 100 \\
0 & \text{otherwise}
\end{cases}$$
The above is easy as $f(x)$ is piecewise linear. Now for $EX$, we could use the formula $\int x f(x) \, dx$ as shown below.

\[
EX = \int x \cdot f(x) \, dx
\]

\[
= \int_{0}^{20} \frac{0.02}{1.1} \cdot x \, dx + \int_{20}^{30} \left[ \frac{0.02}{1.1} - \frac{(x-20) \cdot 0.01}{10} \right] \cdot x \, dx + \int_{30}^{70} \frac{0.01}{1.1} \cdot x \, dx + \int_{70}^{100} \left[ \frac{0.01}{1.1} - \frac{(x-70) \cdot 0.01}{30} \right] \cdot x \, dx
\]

\[
= 4 + \frac{11}{3.11} + \frac{20}{1.1} + \frac{12}{1.1} = 36.06
\]
Solved: We shall use the fact that $E(Be(1/2)) = \frac{1}{3}$. 

$X = 0.4 \text{ Be}(1, 2) \quad \Rightarrow \quad U(0, 20) \triangleq 20 \cdot U(0, 1)$

$0.1/1 \quad \Rightarrow \quad U(0, 30) \triangleq 30 + 10 \cdot U(0, 1)$

$0.05/1 \quad \Rightarrow \quad 20 + 10 \cdot Be(1, 7)$

$0.4/1 \quad \Rightarrow \quad U(30, 70) \triangleq 30 + 70 \cdot U(0, 1)$

$0.15/1 \quad \Rightarrow \quad 70 + 30 \cdot Be(1, 2)$
Hence, 
\[
E X = \frac{0.4 \times 10}{1.1} + \frac{0.1 \times 25}{1.1} + \frac{0.05 \times 70}{1.1} + \frac{0.4 \times 50}{1.1} + \frac{0.15 \times 80}{1.1} = 36.06
\]

The above can be "read off" the graph or using the representation above.

The advantage of this method is phenomenal in terms of insight and also when it comes to computation of higher moments, for example.

Variance. [On. The above has five components — Can you slice and dice differently to come up with only four?] In summary, piecewise linear densities lead to mixtures below. Note that it is \( \text{Be}(2, s) \) and not \( \text{Be}(1, s) \).

Shifted and scaled \( \text{Be}(4, 2), \text{Be}(2, s) \) and \( U(0, 1) \).

The study note would refer to the above as "splicing."