1. Jim is considering different insurance to cover against a random loss $X$, which is distributed as an Exponential with mean 100. His utility $u(w)$ is given by
$$u(w) = -e^{-0.001w} \quad w > 0.$$ 

(a) For an ordinary insurance with deductible $d$, i.e.,
$$I_d(x) = (x-d)^+$$
find the deductible which makes the pure premium equal to $10$.

(b) For a proportional insurance with proportion $k$, find $k$ which makes the pure premium equal to $10$.

(c) For an insurance with an ordinary deductible $D$ and reimbursement only $75\%$ of the loss in excess of the deductible, find $D$ which makes the pure premium equal to $10$.

(d) Which of the two insurances in part (b) or (c) would be preferred by Jim.
\[ E(X-d)_+ = \int_0^\infty (x-d)_+ e^{-0.01x} \, dx \]
\[ = \int d \frac{e^{-0.01(x-d)}}{x} \, dx \]
\[ = e^{-0.01d} \int_0^\infty y \cdot 0.01 e^{-0.01y} \, dy \]
\[ = e^{-0.01d} \times 100 \]

Hence the deductible for a pure premium of $10 will be given by
\[ e^{-0.01d} + 100 = 10 \]
\[ -0.01d = \ln 0.1 \]
\[ d = 230.2585 \]

\[ E(kX) = kEX = k100 \]

Hence \( k \) the proportion in the proportional insurance corresponding to a pure premium of $10 will be
\[ k \times 100 = 10 \]
\[ \text{or} \quad k = 0.1 \text{ or } 10\% \]
\[ I_d^*(X) = \begin{cases} 
0 & X < d \\
0.75 (X - d) & X \geq d 
\end{cases} \] 

\[ E[I_d^*(X)] = \int_{-\infty}^{\infty} 0.75 (x - d) 0.01 e^{-0.01x} dx \]

\[ = \int_{d}^{\infty} 0.75 (x - d) 0.01 e^{-0.01x} dx \]

\[ = 0.75 e^{-0.01d} \int_{0}^{\infty} y 0.01 e^{-0.01y} dy \]

\[ = 0.75 e^{-0.01d} \cdot \text{expectation} \]

\[ = 75 e^{-0.01d}. \]

Hence the deductible in this case is given by

\[ 75 e^{-0.01d} = 10 \]

or \[ d = \frac{-\ln(10)}{-0.01} \approx 201.49 \]
(d) Since it is an expanded utility function, the initial wealth is unimportant, or assume \( w = 10 \).

So the final wealth in the case of proportional insurance will be

\[
10 - X + 0.1X - 10 \uparrow \quad \text{Premium}
\]

and in the case of the deductible + coinsurance it will be

\[
10 - X + 0.75(X - d) - 10
\]

Hence the expected utility of the proportional insurance will be

\[
E \left( U \left( 10 - X + 0.1X - 10 \right) \right) = E \left( U \left( \frac{-99.1}{0.1} \right) \right)
\]

\[
= -\int_0^\infty e^{-0.001(-0.9x)} \cdot 0.01 \cdot e^{-0.01x} \, dx
\]

\[
= -\frac{0.01}{0.9} \int_0^\infty e^{-0.01 - 0.0009} \, dx
\]

\[
= -\frac{0.01}{0.91} = \frac{-1}{1 - 0.09} = \frac{-1}{0.91} \approx -1.0989
\]
The same under the second insurance will be

\[
E \left( U \left( -X + 0.75(X-d)_+ \right) \right)
\]

\[
= \int_{-\infty}^{\infty} e^{-0.001(-x + 0.75(x-d)_+)} d\frac{-0.001}{0.01} e^{0.01 x} dx
\]

\[
= \int_{0}^{d} e^{-0.001(-x)} 0.01e^{-0.01x} dx + \int_{d}^{\infty} e^{-0.001(-x + 0.75(x-d)_+)} 0.01 e^{-0.01x} dx
\]

\[
= \int_{0}^{d} e^{-0.01} e^{(0.01-0.001)x} dx + \int_{d}^{\infty} e^{-0.00075d} 0.01 e^{-0.01x} dx
\]

\[
= \left[ \frac{0.01}{0.01 - 0.001} \left[ 1 - e^{-(0.009)d} \right] + e^{-0.00075d} 0.01 e^{-0.00975d} \right] - \frac{0.09298 + 0.12369}{0.00975}
\]

\[-1.053 > -1.0989\]

Hence the insurance with the deductible and coinsurance will be preferred to the proportional insurance.