Instructions

i. This assignment contains some problems in elementary probability to give you an idea of the probabilistic pre-requisites for this course.

ii. Parts carrying extra credits and starred problems are not required part of the assignment. Nevertheless, a serious attempt followed by a discussion of these during office hours is encouraged.

iii. Constructive comments on the assignments and for that matter any other aspect of the course will be welcomed.

Problem 1

Consider a decision maker with an exponential utility function given by

\[ u_\alpha(w) = -\exp(-\alpha w), \quad \forall w \]

Let \( G_\alpha \) be the break-even premium or net premium that the decision maker is willing to pay for full insurance against a possible loss modeled by the random variable \( X \). In class we used the moment generating function to show that

\[ \lim_{\alpha \to 0} G_\alpha = E(X) \]

Here we give a reason to expect the above to happen; to this end, do the following problems.

i. Show that the net premium is the same with respect to the modified utility function, say \( u'(\cdot) \), given by

\[ u'_\alpha(w) = \frac{1 - \exp(-\alpha w)}{\alpha}, \quad \forall w \]

ii. Find the limiting utility function defined as \( \lim_{\alpha \to 0} u'_\alpha(\cdot) \) - Show that it is a linear utility function.

iii. Confirm that the expectation is the net premium with respect to any linear utility function.

The following problems are from the text, *Actuarial Mathematics*: 1.2 part d, 1.13, 1.14, 1.18 and 1.19.

Problems 1.21, 1.22 and 1.23 all carry extra credits - I would add that in my opinion they are a "must do" for all passionate actuaries.