Problem Solving 7

Lecture 18 May 16, 2021

- Q1. C_1 and C_2 are two circles of radii 3 and 1, respectively, such that the distance between their centers is 10. Let S be the set of points in the plane that are the middle point of a line segment with one end on C_1 and the other end on C_2.

What is the area of S?

- Q2. Consider the sequence defined by $a_{n+1} = 2a_n + 5$ and $a_1 = 1$.
- Which of the following numbers will appear in this sequence

- 562301
- 786427
- 16485
- 3123
- 51519

- Q3. In a factory, every worker is friend with some other workers and his/her salary is equal to the average of the salary of his/her friends. Which of the followings is correct:

- There is one worker whose salary is equal to the average of the salary of all others.
- No one earns more than twice of any other person.
- All the salaries are the same
- Every two friends earn the same
- None

- Q4. Suppose x, y, z are 3 real numbers such that $xyz(x + y + z) = 1$.
- What is the minimum possible value of $(x + y)(y + z)$?

- Q5. Let M be a point inside the triangle ABC. Which of the following statements about the value of |AM|+|BM|+|CM| is always true:

- It is always smaller then the largest side of ABC
- It is always smaller than the sum of the largest two sides of ABC
- It is always more than the sum of the smallest two sides of ABC
- It is always larger than 3 times the radius of circumscribed circle of ABC
- It is always smaller then the sum of the two largest heights of ABC

- Q6. Suppose that the numbers $a_1, a_2, \ldots, a_{2021}$ are a permutation of 1,2, …, 2021.

Let $L = |a_1 - 1| \times |a_2 - 2| \times \cdots \times |a_{2021} - 2021|$.

Which numbers between 1 to 10 can appear as a value of L?

- Q7. Suppose a_1 is a natural number. For $n > 1$ define a_{n+1} to be the largest prime number dividing a_n+1. We say a_1 is good if the sequence $a_1, a_2, a_3, \ldots$ is eventually periodic. Which of the following statements is correct:

- There are only finitely many good numbers.
- There are infinitely many bad numbers
- All numbers are good
- All numbers are bad
- There are bad numbers, but only finitely many