Quiz 5, Name: Key__________________________

1.Find maxima and maximal value of the function $2x^2 - x + y^2$ in the half disk

$x^2 + y^2 \leq 1$ and $x \geq 0$.

Let $f(x, y) = 2x^2 - x + y^2$.

a.Then $f_x = 4x - 1 = f_y = 2y = 0$ will imply $(1/4, 0)$ is the only interior critical point.

b. On the semi-circle, we take $g(x, y) = x^2 + y^2 = 1$. By the Lagrange multiplier,

There is a λ so that

$$\left\{ \begin{array}{ll} 4x - 1 & = \lambda 2x \\ 2y & = \lambda 2y \\ x^2 + y^2 & = 1 \end{array} \right\}$$

which leads to points $(1, 0)$ and $(1/2, \pm\sqrt{3}/4)$. We should also include the points on the ends of the semi-circle $(0, \pm 1)$.

Now let's look at the other part of the boundary, namely the interval on the $y-$axis. Here we have $g(x, y) = x = 0$.

Hence we have

$$\left\{ \begin{array}{ll} 4x - 1 & = \lambda \\ 2y & = 0 \\ x & = 0 \end{array} \right\}$$

which leads to one point $(0, 0)$. By comparing values at these seven points, one can see that the maxima are at $(0, \pm 1)$ and $(1, 0)$ with maximum value 1.

2. Find the integral $\int\int\int_D x dvol$, where D is the part of the unit ball $x^2 + y^2 + z^2 \leq 1$ and $x \geq 0, y \geq 0, z \geq 0$.

We will use spherical coordinates:

$$\int\int\int_D x dvol = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho \sin\phi\cos\theta\rho^2 \sin\phi d\rho d\phi d\theta$$
$$= \frac{1}{4}\frac{\pi}{4} = \frac{\pi}{16}.$$