Math42, Quiz3 Date: _______October ____30

We will use the triange $\Delta : x \geq 0$, $y \geq 0$,and $x+y \leq 1$.

1. Find the extrema of the function x^2y.in the triangle Δ.
2. Compute $\int\int_\Delta x^2ydxdy$.

Solution:1.
$f(x,y) = x^2y$.
Then $f_x = 2xy, f_y = x^2$.Hence $f_x = f_y = 0$ implies $x = 0$ and for all y.
Namely all point of the form $(0,y)$ but they are on the boundary. So no interior critical points.

The boundary has three parts. However it is easy to observe that
$f = 0$ on $x = 0$ and $y = 0$ so we don't have to do calculus there.
On the part that $g(x,y) = x+y = 1$, we use Lagrange multiplier:

$$f_x = 2xy \quad = \lambda g_x = \lambda$$
$$f_y = x^2 \quad = \lambda g_y = \lambda$$

We have $2xy = x^2$ and therefore $x = 0$ (which is on the boundary) and $2y = x$ with the condition
$x+y = 1$ will imply $x = 2/3, y = 1/3$.
Finally the list of possible extrema, $(0,y), (x,0), (2/3,1/3)$.
One can see that $f = 0$,which is minimum value, on the axis and $f(2/3,1/3) = 4/27$,which is maximum value.

2. $\int\int_\Delta x^2ydxdy = \int_0^1 \int_0^{1-y} x^2ydxdy$
$= \int_0^1 \frac{1}{3}x^3y|_0^{1-y}dy = \int_0^1 \frac{1}{3}(1-y)^3ydy = \int_0^1 (\frac{1}{3}(1-y)^3 - \frac{1}{3}(1-y)^4)dy = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$.