Assignment 10

1. One of the characteristics of leukemia is an excess of white blood cells. The white blood cell count at diagnosis can be used to aid in predicting a patient's survival time after diagnosis, with high white blood cell counts indicating a low expected survival time. Feigl and Zelen (Biometrics, 1965) show survival times in weeks and white blood cell counts (WBC) at diagnosis for 33 patients who died of acute leukemia. The patients were classified as AG positive or AG negative depending on the presence or absence of certain characteristics in the white blood cells. In this problem consider the data for the 17 patients classified as AG positive. The data set is also available online or the full data is available as `leuk` in the `MASS` package.

   Feigl and Zelen model the conditional distribution of the lifetimes given the white blood cell count as exponentially distributed with mean

   $$\theta_1 \exp(-\theta_2 x_i),$$

   where

   $$x_i = \log(\text{WBC}_i/10000)$$

   The maximum likelihood estimates for this model are $\hat{\theta}_1 = 56.85$ and $\hat{\theta}_2 = 0.482$.

   Consider a proper but vague prior distribution with θ_1 and θ_2 independent a priori and

   $$1/\theta_1 \sim \text{Exponential with mean } 1000$$
   $$\theta_2 \sim \text{Uniform on } [0, 1000].$$

   (a) Verify the maximum likelihood estimates graphically by plotting the log likelihood function and numerically using a nonlinear optimizer such as `optim` in R.

   (b) Compute the unnormalized marginal posterior density of θ_2 by analytically integrating out θ_1 and plot the resulting unnormalized density.

   (c) Use either the rejection sampling or the ratio-of-uniforms approach to construct a method to sample from the marginal posterior density of θ_2. Use graphical, numerical, or analytical methods to choose a reasonable bound for rejection sampling or the shift μ and to choose bounding rectangle for ration-of-uniforms sampling. Generate a sample of size 10000 and graphically compare a density estimate for your sample to the analytic form of the unnormalized density.

   (d) Augment your sample of θ_2 values to a sample of θ_1, θ_2 pairs by drawing θ_1 values from the appropriate conditional distribution.

(e) Use your sample to construct estimates of the posterior means and variances of θ_1 and θ_2, the posterior covariance of θ_1 and θ_2, and the marginal posterior density of θ_1. Obtain standard errors for your moment estimates and provide a plot of the approximate marginal posterior density of θ_1. Use any variance reduction methods that seem appropriate.

You should submit your assignment electronically using Icon. Submit your work as a single compressed tar file. If your work is in a directory `mywork` then you can create a compressed tar file with the command

```
tar czf mywork.tar.gz mywork
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