22M174/22C174: Optimization techniques.

Homework 7. Due 04/10/13.

1. Is the function $f(x) := 2x^3$ convex on $[-2, 2]$? Same question for the function $g(x) := -2x^3$ on $[-4, -2]$.

2. Prove that the set $C := \{[x_1, x_2]^T \in \mathbb{R}^2 \mid 3x_1 - 2x_2 > 0\}$ is convex. Is the function $f(x_1, x_2) := x_1^4 + x_2^2 - \ln(3x_1 - 2x_2) + |2x_1 - 3|$ convex on C?

3. Assume that $f : C \subset \mathbb{R}^n \to \mathbb{R}$ is convex on a convex set C. Let $x^{(1)}, \ldots, x^{(m)}$ be arbitrary points of C and $\alpha_1, \ldots, \alpha_m$ be nonnegative numbers satisfying $\sum_{i=1}^m \alpha_i = 1$. Show that

$$f\left(\sum_{i=1}^{m} \alpha_i x^{(i)}\right) \leq \sum_{i=1}^{m} \alpha_i f\left(x^{(i)}\right).$$

   Hint: use induction on m.

4. Find the global minimizers and maximizers of $f(x_1, x_2) := x_1 x_2$ in the set

$$X := \{x \in \mathbb{R}^2 \mid x_1^2 + 4x_2^2 = 4\}$$

   first without using Lagrange multipliers (i.e., by elimination) and then by using Lagrange multipliers.

5. Using Lagrange multipliers, find the global minimizers and maximizers of $f(x_1, x_2, x_3) := x_3 - x_2 - x_1$ in the set

$$X := \{x \in \mathbb{R}^3 \mid x_3^2 + 2x_2^2 = 1,\ 3x_3 - 4x_1 = 0\}.$$