Algorithmic Excursions (CS:4980:0001 or 22C:196:001)
Homework 1

The homework is due in class on Thursday, February 25th. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

1. In lecture notes for Week 2, complete the proof of **Lemma 3.1** by showing that $|N|$ is $O(\frac{1}{\epsilon} \ln |\mathcal{R}|)$.

2. In lecture notes for Week 2, prove **Lemma 3.9**. Assume that $0 \le \epsilon + \epsilon' \le 1$.

3. In lecture notes for Week 3, we proved **Theorem 3.1** in the special case that $|X|$ is an integer power of 2. Prove the theorem for the general case, by reducing to the special case.

4. In lecture notes for Week 3, prove **Claim 3.2**.

5. In lecture notes for Week 3, prove **Claim 3.3** using **Claim 3.2**, **Theorem 3.1**, and from the notes for Week 2, **Lemma 3.1**.

6. In lecture notes for Week 3, answer the question embedded in the proof of **Claim 3.5**. For your convenience, here is the question:

   Let $Y_1, Y_2, \ldots, Y_s$ be independent 0-1 random variables, where $\Pr[Y_i = 1] \ge \frac{1}{r}$ for each i, where $r \ge 2$. Let $Y = \sum_{i=1}^{s} Y_i$. Note that $E[Y] \ge \frac{s}{r}$. Using Chebyshev's inequality, and assuming $\frac{s}{r}$ is larger than some absolute constant, show that $\Pr[Y \le \frac{s}{2r}] < 1/2$. You will need to upper bound the variance of Y for this.