22C : 031 (CS : 3330 : 0001) Algorithms

Homework 2

February 1, 2012

This homework is based on our discussion of the greedy algorithm for scheduling to minimize lateness, and our discussion of graph algorithms. The homework is worth 10 points.

1. A company is interviewing n candidates on a particular day. Each candidate needs to first have a face-to-face meeting with a panel, and then take a written exam. Since the candidates are from different backgrounds, the company has planned meetings and exams of different lengths for the candidates. Let m_i and e_i denote the lengths, in minutes, of the face-to-face meeting and the written exam, respectively, for candidate i.

   (We are assuming that the candidates are $\{1, 2, \ldots, n\}$.)

   Since there is only one panel, the face-to-face meetings cannot overlap. However, the written exams can be taken simultaneously by any number of candidates. So the company decides to schedule the face-to-face meetings in a certain order starting at 8:00 am. Each candidate proceeds to take her written exam immediately after her meeting.

   Describe a polynomial time algorithm that takes as input the m_i and the e_i values for each of the n candidates and computes an ordering of the candidates (for the face-to-face meetings) that results in the entire process completing as early as possible. (The process is said to complete when every candidate has finished their written exam.) (2.5 points)

2. We are given a set of n jobs – the i-th job has a processing time $t_i > 0$ and a weight $w_i > 0$. We want to order the jobs so as to minimze the weighted sum of completion times, $\sum_{i=1}^{n} w_i C_i$.

   The completion time C_i of job i in a given ordering of the jobs is the sum of the processing times of all jobs up to and including i in the order. For example, for the ordering $\langle 2, 3, 1 \rangle$, the completion time of job 2 is t_2, of job 3 is $t_2 + t_3$, and of job 1 is $t_2 + t_3 + t_1$.

   Design an efficient and correct greedy algorithm for the problem.

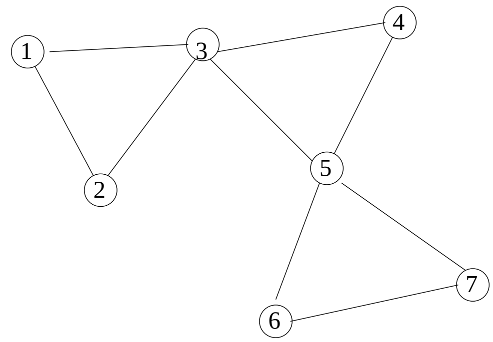

Figure 1: Graph for Problem 3

Argue that the greedy algorithm works correctly whenever $n = 2$, that is, for all inputs with two jobs. (3.5 points)

3. Show the breadth-first-search (BFS) tree on the graph in the figure when a BFS is run from node 1. You can assume an arbitrary ordering within the adjacency list of each node of the graph, but fix and specify the ordering in advance. (1.5 points)

4. A native Australian named Anatjari wishes to get from point A to point B across a desert carrying only a single water bottle. He has a map that marks all the watering holes in the desert. Assuming he can walk k miles on one bottle of water, design an efficient algorithm for determining the sequence of watering holes that he should visit in order to make as few stops as possible. We can assume that he starts off with a filled bottle.

   (For concreteness, assume we are given the x and y coordinates of each watering hole, the coordinates of the start location, and the coordinates of the end location. In addition, we are given the number k. We can assume that it is possible to walk the straight line path between any two locations, given enough water.) (2.5 points)

The homework is due Wednesday, February 8, in class; if you can't make it to class on that day, just make sure you get it to me by that time.